Market Expectations, Term Premia, and the Short Rate in the Term Structure of Interest Rates

XIAOXIA YE*

September 2, 2013

School of Business, Stockholm University

Abstract

This paper develops a novel approach to legitimately measure the market expectations of future changes of the yield curve. By-products of this approach are generic impact measures of state variables. Unlike loading coefficients of state variables, these measures are unique to a particular state variable regardless of how other state variables are defined within the model. This uniqueness feature is used to set up constraint conditions in model parameter calibration such that the resulting measures of market expectations best predict the future changes while the corresponding term premia have the legitimate impact on the forward curve. Based on a three factor Gaussian dynamic term structure model and the calibrated model parameters, I then show the generic impact of the short rate on the yield curve, and the historical dynamics of market expectations and term premia, using the U.S. treasure yield data. Three case studies of recent unconventional monetary policies are also included.

Keywords: term structure of interest rates, market expectations, short rate, LSAP, MEP, QE3
JEL classification: E43

*School of Business, Stockholm University, S-106 91 Stockholm Sweden (email: xiye@fek.su.se). I thank the participants at WB/BIS Joint Fourth Public Investors' Conference. All errors are mine.

1 Introduction

Correctly measuring the market expectations about the future yield curve is a topic of great concerns to bond traders and monetary policy-makers alike. A good understanding on how the yield curve movement follows the changes in the market expectations can be the key to a successful trading strategy. Policy-makers monitor the market expectations constantly, a detailed knowledge about the impact of changes in the expectations is crucial for them to set up effective monetary policies. And the market expectations are the key to estimating the term premia, they are two sides of the same coin.

There have been studies on how to extract the information about market expectations from the interest rate related instruments, for example: Söderlind and Svensson (1997); Brooke et al. (2000); Joyce et al. (2008). However, these studies do not explicitly remove the term premia component embedded in the implied forward rate curve. They basically measure the risk-neutral expectations. As we all know the term premia are not constant, and they will even be very volatile some times, especially when market uncertainty is high. Therefore the risk-neutral expectation should not be regarded as a good measure of the market expectation as the changes in risk-neutral expectation might be due to changes in risk premia instead of the market expectation.

Kim and Wright (2005) use a three factor Gaussian dynamic term structure model to estimate both market expectations and term premia. Their approach is similar to mine. However, they do not deal with the bias in model parameter ($\mathbb{P}$-measure) estimation. As I show in this paper, the state variable $\mathbb{P}$-measure dynamics implied by the ML estimates of the $\mathbb{P}$-measure parameters have little predicting power for the future changes of the yield curve, and they predict the future much worse than do the $\mathbb{Q}$-measure dynamics. So the resulting measure of market expectations are far from being legitimate.

Adrian et al. (2013) recently develop a new estimation approach for term structure models in which all parameters are estimated by linear regressions. In their paper, the authors do calibrate the market price of risk parameters such that the estimates give best predicting power for one month excess holding returns across different maturities. However, their calibration is limited to only one month forecasting horizon while the approach proposed in this paper consider the forecasting horizon as long as five years. And as shown later on, the parameters calibrated to shorter horizon prediction tend to perform much worse in longer horizon prediction due to the over-fitting problem and inflexible specification of the market prices of risks. And the over-fitting problem might result in non legitimate measures of market expectation and term premia. Another difference is that their models are discrete time while my approach here is based on a continuous time model which requires no extra efforts when fitting to data of different frequency.

Cochrane and Piazzesi (2005, 2008) develop models in which one of the state variables is designed to have high predicting power for one year excess holding returns. Therefore, their models are calibrated to best forecast the changes in one year, again might run into over-fitting problem. They do not look at the impact of their term premia measures on the forward curve which is rigorously inspected in this paper, as a high R^2 does not necessary represent a legitimate measure of term premia. One more difference is that their models are also discrete-time.

Due to the difficulties in estimating market expectations using model-based approaches, some researchers are resorting to usage of survey-based data to analyze the market expectations, see, e.g., Kim and Orphanides (2012); Piazzesi et al. (2013). However, survey-based data are very different than the yield data, and the latter constantly reflects the prices of the risks faced by bond market participants, and the survey-based expectations might suffer seriously from having poor predicting power. As mentioned in Cook and Hahn (1990): "the forecasters used in the survey might be influenced by the current shape of the yield curve in determining their interest rate forecasts, in which case the regression results would be spurious. Or they might systematically differ in their forecasts from the market in general for other reasons, perhaps because their forecasts are made public or because they are not actively involved in buying and selling securities." Therefore, the approach proposed here is purely model- and yield data-based.

The paper starts from estimating a 3-factor Gaussian dynamic term structure model using the U.S. treasure yield data. Then in a regression exercise, future changes of yields are regressed onto the current state variables. I find that in these unrestricted linear regressions state variables are able to predict the future changes decently well especially when the forecasting horizon is around 3 to 4 years (the R^2 for predicting the 3mth yield change in 3.5 years can be as high as 85%). By assuming constant market prices of risks, the risk-neutral expectations predict the changes worse than do the unrestricted state variables, but still have reasonable predict power. However, when it comes to the model implied (using the ML estimates of market prices of risks parameters) expected changes of the yields, I find that they have much worse predicting power for shorter maturity yields and no predicting power at all for longer maturity yields. This just confirms the findings about the bias of ML estimates of the $\mathbb{P}$-measure parameters shown in Duffee and Stanton (2012) and Bauer et al. (2012).

Therefore in order to capture the empirical feature (high R^2s in predicting the future changes of yields) of the state variables, we have to calibrate the parameters. Fortunately, the ML estimates of $\mathbb{Q}$-measure parameters are unbiased as shown in Duffee and Stanton (2012) and many others. Therefore only the $\mathbb{P}$-measure parameters need to be calibrated. I then calibrate the $\mathbb{P}$-measure parameters such that the model implied expected changes of the yields predict the realized changes of yields as well as possible (in a least square sense).

In order to check if the resulting expectation measures have the correct impact on the yield curve, i.e., to check the legitimation of the measure, I develop a *(un)conditional contemporaneous impact* (UCI and CCI) measure to exam the impact of the expectations on yield curve. Unlike loading coefficients of state variables, these measures are unique to a particular state variable regardless of how other state variables are defined within the model. And I find that the calibrated $\mathbb{P}$-measure parameters result in nonsensible term premia which have a negative impact on forward rates given a positive shock. I therefore conclude that the $\mathbb{P}$-measure parameters calibrated using unconstraint optimization do not give rise to legitimate measure of market expectations. Using the UCI measure, I define a constraint optimization such that the expectation and term premia measures constructed from the calibrated $\mathbb{P}$-measure parameters have the correct UCI on the forward curve. These constraint parameters give more evenly distributed R^2 curves across maturities and forecasting horizons than the unconstraint parameters, and their R^2 curves outperform their lower bounds at shorter maturities and get close to them at longer maturities.

Using UCI and *unconditional persistent impact* (UPI) measures, I show show the generic impact of the short rate on the yield curve. I find that a significant portion (40%) of a short rate shock has a uniform impact on the middle part and the long end of the yield curve, and the impact is also quite persistent (at least over 4 years of time). Using the constraint parameters, I construct and show the historical dynamics of market expectations and term premia. I find that in the period of the conundrum during 2004-05 the drop in term premia not only happened at long maturities but also at short maturities although the short maturity yield was increasing during that period; after late of 2010 the short maturity yield mainly consisted of the term premium, and it has been shrinking since mid of 2011 as the market expectation has been increasing and short maturity yield kept decreasing.

In the case studies, I analyze the impacts of the announcement of the large scale asset purchase program (LSAP), the maturity extension program (MEP), and the third round of Quantitative Easing (QE3) on the market expectations and term premia, and also present the *conditional persistent impact* (CPI) of these announcement up to a 4 years of time. Upon the announcements of the LSAP and MEP, although the zero yield curve moved in line with the intents of the Fed, the market responded differently to these two announcements: the market responded clearly and positively to the LSAP by lowering the average expected short rate in most of the maturities, the extents are larger for longer maturities; however, it responded less so to the MEP by increasing the average expected short rate at maturities shorter than 6-yr and decreasing it at maturities longer than 6-yr by smaller extents. Upon the announcements of the QE3, the forward curve moved against the intent of the Fed, i.e., the long-end of the forward curve increased when the QE3 was announced. The average expected short rate decreased at short and middle maturities, but it consistently increased at long maturities. From the CPI measures, I find that the LSAP had a significant impact on lowering the whole yield curve in the long run; but the long run impact of MEP on lowering the yield curve was much less prominent; when it comes to the QE3, the long run impact was actually to push up the whole yield curve.

The rest of the paper is organized as follows. Section 2 describes the model and how it is estimated. Section 3 defines UCI, CCI, CPI, and UPI. Section 4 discusses the calibration of $\mathbb{P}$-measure parameters. Section 5 and Section 6 present the empirical results and case studies. Finally, Section 7 concludes the paper. Appendix includes technical details.

2 Modeling and Estimation

In this section, I apply the modeling framework recently developed by Li and Ye (2012) (hereafter LY) to set up the 3-factor Gaussian dynamic term structure model.

2.1 Base Model

The LY framework is HJM-based, the starting point is the instantaneous forward rate. Following the Musiela parametrization (Brace et al., 1997), the time-t instantaneous forward rate for time-$t+x$ is written as:

$$\begin{aligned} r(t,x) &= r(0,t+x)+\Theta(t,x)+r_0(t,x) \\ \Theta(t,x) &= \int_0^t \sigma(x+t-s)\int_0^{x+t-s}\sigma(v)\,dv\,ds \\ r_0(t,x) &= \int_0^t \sigma(x+t-s)\,dW_s \end{aligned} \tag{2.1}$$

where W_t is a 3-dimensional $\mathbb{Q}$-measure Brownian motion, $\sigma(\bullet)\in\mathbb{R}^3$ is the time-invariant volatility function defined in the following SDE:

$$dr(t,x)=\frac{\partial}{\partial x}r(t,x)+\sigma(x)\int_0^x \sigma(s)^{\intercal}ds+\sigma(x)\,dW_t.$$

The key ingredient under LY's framework is the volatility function. Here I specify the volatility function to be consistent with the most classic 3-factor Gaussian dynamic model, $AM_0(3)$ due to Dai and Singleton (2000).

$$\sigma(x)_{1\times 3}=\begin{bmatrix} e^{-k_1x} & e^{-k_2x} & e^{-k_3x}\end{bmatrix}\begin{bmatrix}\Omega_1 & 0 & 0\\ \Omega_2 & \Omega_4 & 0\\ \Omega_3 & \Omega_5 & \Omega_6\end{bmatrix}.$$

By the definition, if $\sigma(x)$ is factored as $\mathbf{C}\exp(\mathbf{A}x)\mathbf{B}$, then $r_0(t,x)$ has the following Markov representation:

$$\begin{aligned} r_0(t,x) &= \mathbf{C}(x)Z_t \\ dZ_t &= \mathbf{A}Z_t dt+\mathbf{B}dW_t, \quad Z_0=0, \end{aligned}$$

where $\mathbf{C}(x)=\mathbf{C}\exp(\mathbf{A}x)$.

Following LY, I set $r(0,t+x)+\Theta(t,x)$ to its time-homogeneous counterpart $\varphi+$

$\Theta^*(x)$, where

$$\Theta^*(x) = \mathbf{C}(x)\left(\mathbf{A}^{-1}\mathbf{B}\mathbf{B}^\intercal(\mathbf{A}^\intercal)^{-1}\right)\left(\mathbf{C}(0)^\intercal - \frac{\mathbf{C}(x)^\intercal}{2}\right).$$

Using the modified "essentially affine" (Duffee, 2002) market price of risk setting proposed by LY, the $\mathbb{P}$-measure dynamic of Z_t is

$$dZ_t = \mathbf{A}^{\mathbb{P}}(Z_t - \mu)\,dt + \mathbf{B}dW_t^{\mathbb{P}}, \tag{2.2}$$

where

$$\mathbf{A}^{\mathbb{P}} = \mathbf{A} - \mathbf{B}\underbrace{R_m}_{3\times 3}\mathbf{C}\left(\underbrace{mV}_{3\times 1}\right), \mu = \left(\mathbf{A}^{\mathbb{P}}\right)^{-1}\mathbf{B}\underbrace{R_v}_{3\times 1}$$

and

$$W_t^{\mathbb{P}} = [R_v + R_m\mathbf{C}(mV)Z_t]\,t + W_t.$$

mV is a 3-dimensional vector of maturities,[1] $W_t^{\mathbb{P}}$ is a 3-dimensional $\mathbb{P}$-measure Brownian motion.

There are infinitely many representations, as given one realization $\{\mathbf{A}, \mathbf{B}, \mathbf{C}(x)\}$, for any invertible matrix M, $\left\{M\mathbf{A}M^{-1}, M\mathbf{B}, \mathbf{C}(x)M^{-1}\right\}$ is another realization, since

$$\mathbf{C}(0)\exp(\mathbf{A}x)\mathbf{B} = \mathbf{C}(0)M^{-1}\exp\left(M\mathbf{A}M^{-1}x\right)M\mathbf{B} = \mathbf{C}(x)M^{-1}M\mathbf{B}.$$

The realization employed by Dai and Singleton (2000) is only one of them, the details are presented in LY. In this paper, I first consider the "Base realization". Since this is a Jordan form realization, I use this realization to estimate parameters. Then given the parameter estimates, I rotate the state variables to construct different realizations in which state variables are of particular economic meaning, say, e.g., the market expectation of future change of yields, and term premia. The rotation is done by defining different transfer matrices M.

[1] mV is chosen to be $\begin{bmatrix}\text{3mth}\\ \text{2yr}\\ \text{10yr}\end{bmatrix}$ in the empirical analysis.

In the “Base realization”, the risk neutral parameter triplet is given as follows:

$$
\begin{aligned}
\mathbf{A}_{\text{Base}} &= \begin{bmatrix} -k_1 & 0 & 0 \\ 0 & -k_2 & 0 \\ 0 & 0 & -k_3 \end{bmatrix} \\
\mathbf{B}_{\text{Base}} &= \begin{bmatrix} \Omega_1 & 0 & 0 \\ \Omega_2 & \Omega_4 & 0 \\ \Omega_3 & \Omega_5 & \Omega_6 \end{bmatrix} \\
\mathbf{C}(x)_{\text{Base}} &= \begin{bmatrix} e^{-k_1 x} & e^{-k_2 x} & e^{-k_3 x} \end{bmatrix}.
\end{aligned}
$$

2.2 Model estimation

The data used for estimation are the U.S. Treasury constant maturities yield curve rates, downloaded from Federal Reserve Statistical Release, H.15, Selected Interest Rates (Daily). The data-set contains daily observations of the yields at maturities of 3, 6-month, 1, 2, 3, 5, 7, and 10-year. They span the period from January 1994 to October 2012. As the Treasury yield curve is considered as the par curve, the par curves are converted to the zero curves by first smoothing par rates then bootstrapping the zero rates from par rates.

The parameters are estimated using Kalman filter in conjunction with QMLE, assuming IID normal measurement errors. The estimates of $\mathbb{Q}$-measure parameters are presented in Table 2.1. The summary statistics of the pricing errors are shown in Table 2.2. Unsurprisingly, the 3-factor Gaussian model does a good job in fitting the dynamic term structure of interest rates. The average MAE (mean absolute error) across maturities is only 4.2bp, and the average VR (variance ratio) across maturities is as high as 99.9%. This is consistent with the previous literature on the fitting performance of 3-factor Gaussian dynamic term structure models, e.g., Heidari and Wu (2009).

2.3 Bias of the ML estimates of the $\mathbb{P}$-measure parameters

Here I do not report the ML estimates for R_v and R_m. The reason is that they are strongly biased. This has been well documented in the literature. One important reference is Duffee and Stanton (2012) who use intensive Monte Carlo experiments to show that

Table 2.1: Parameter Estimates

$$\mathbf{A}_{\text{Base}} = \begin{bmatrix} \underset{(0.004)}{-0.1509} & 0 & 0 \\ 0 & \underset{(0.009)}{-0.3255} & 0 \\ 0 & 0 & \underset{(0.047)}{-1.3647} \end{bmatrix}, \mathbf{B}_{\text{Base}} = \begin{bmatrix} \underset{(0.004)}{0.0478} & 0 & 0 \\ \underset{(0.004)}{-0.0528} & \underset{(0.001)}{0.0151} & 0 \\ \underset{(0.001)}{0.0065} & \underset{(0.001)}{-0.0151} & \underset{(0.000)}{0.0068} \end{bmatrix}$$

$$\varphi = \underset{(0.000)}{0.0551}$$

This table reports parameter estimates using the Base realization. The standard errors are in parentheses. The sample is daily from January 1994 to October 2012

MLE (or in conjunction with Kalman filter) is not able to give unbiased estimates of the parameters related to the market price of risk. Although Bauer et al. (2012) do provide a new estimation method which somewhat corrects the bias of the estimates, the gap between the true parameters and estimated ones is still big as found in their Monte Carlo simulations. Despite the bias of estimates the $\mathbb{P}$-measure parameters, MLE does recover the unbiased estimates of the $\mathbb{Q}$-measure parameters.

In this paper, I calibrate the market price of risk parameters using a novel approach such that the model implied expected changes of the short rate consistently represent the market expectations. And the calibrated $\mathbb{P}$-measure parameters better match the empirical observations. However, I do not claim that the estimates are unbiased. Statistical properties of this calibration will be rigorously explored soon in future research.

Before presenting the calibration, let me first introduce important measures of a state variable's impact on the yield curve which will be used to access the legitimation of the model implied "market expectations".

Table 2.2: Summary Statistics of Pricing Errors

Maturity	Mean(bp)	Medn(bp)	Std.(bp)	MAE(bp)	Max(%)	Min(%)	VR(%)
3m	-1.929	-1.517	6.302	4.384	0.235	-0.784	99.912
6m	2.464	2.097	4.908	4.020	0.498	-0.147	99.948
1y	-0.004	0.254	6.609	4.772	0.415	-0.236	99.904
2y	0.952	1.168	4.050	3.347	0.148	-0.236	99.962
3y	-2.020	-1.584	4.071	3.215	0.186	-0.297	99.958
5y	-1.266	-0.995	4.611	3.775	0.201	-0.178	99.931
7y	3.107	2.203	5.256	4.472	0.235	-0.115	99.889
10y	-1.314	-1.343	6.610	5.219	0.239	-0.207	99.763
Ave.	-0.001	0.035	5.302	4.150	0.270	-0.275	99.908

This table reports the summary statistics (sample mean, median, standard deviation, mean absolute error, maximum, and minimum) of the pricing errors over the daily sample from January 1994 to October 2012. The pricing error is defined as the difference between the market observation and the model implied value. The table also reports the variance ratio (VR) at each maturity, which is defined as one minus the ratio of the pricing error variance to the variance of the original series.

3 Yields Responses

3.1 Contemporaneous Impact

3.1.1 A Misuse of the Loading Coefficients

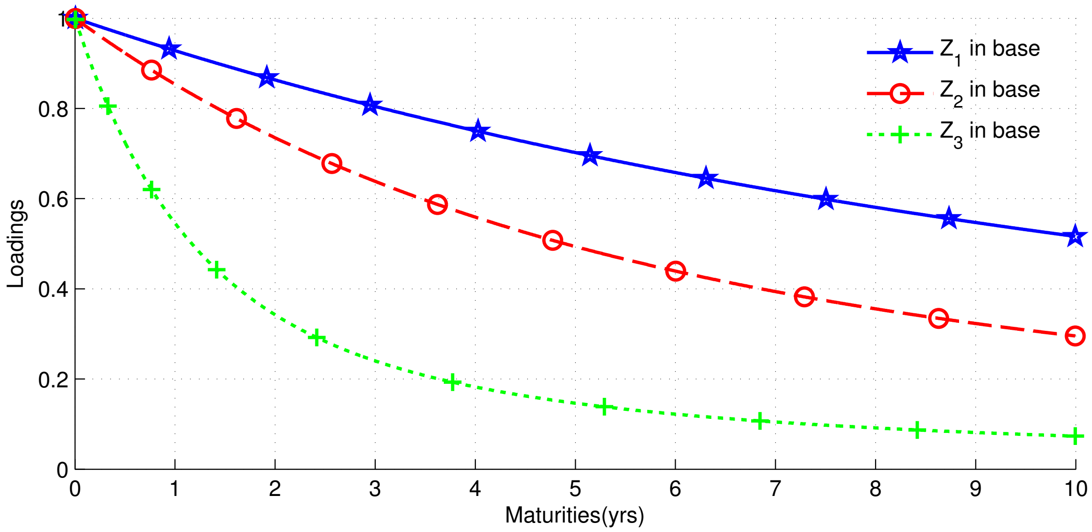
Given (2.1), the zero yield with time-to-maturity of x can be written as

$$y_t(x) = \varphi + \frac{\int_0^x \Theta^*(s)\,ds}{x} + \frac{\int_0^x \mathbf{C}(s)\,ds}{x} Z_t.$$

The loading coefficients of the state variables are $\frac{\int_0^x \mathbf{C}(s)ds}{x}$, their estimates at different maturities are plotted in Figure 3.1. In the term structure literature, these estimates are often used to measure the contemporaneous response of the yield curve to a shock in a state variable, see, e.g., Ang and Piazzesi (2003), Wu and Zhang (2008), Cochrane and Piazzesi (2008), and Lu and Wu (2009). However, this is a misuse of the loading coefficients. The interpretation based on the contemporaneous responses measured by the loading coefficients can be seriously misleading.

Figure 3.1: Loading Coefficients against Maturities

This figure plots the loading coefficients $\frac{\int_0^x \mathbf{C}(s)ds}{x}$ of the three state variables in the "Base realization" against maturities from 1-day to 10-year.



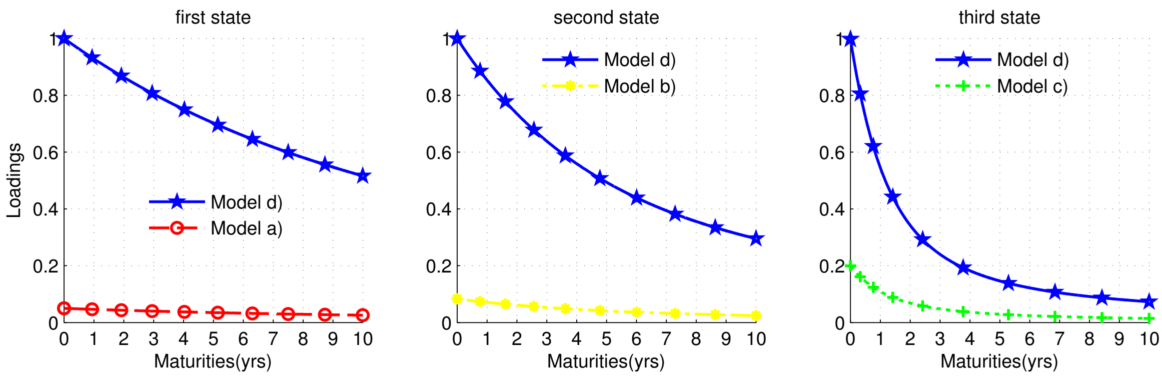
Each one of the curves in Figure 3.1 only presents the responses of the yield curve to a change in the state associated with this curve while other two states do NOT change. Therefore the shape of the curve very much depends on how all three state variables are constructed especially when they are correlated. To visualize the point, let's first arbitrarily define a transfer matrix

$$M = \begin{bmatrix} 20 & 1 & 22 \\ 9 & 12 & 23 \\ 27 & 11 & 5 \end{bmatrix},$$

and denote the new model with states being MZ_t by model d). Then consider the following three models: a) a model with the first state being the first state in model d and the other two being the second and third states of the base model; b) a model with the second state being the second state in model d and the other two being the first and third states of the base model; c) a model with the third state being the third state in model d and the other two being the first and second states of the base model. The loading coefficients of each state variable in two different models are plotted in Figure 3.2. We can

Figure 3.2: Loading Coefficients in Different Models

This figure plots the loading coefficients (from 1 day to 10 years) of the same state variable in two different models: model a) and model d) have the same first state variable; model b) and model d) have the same second state variable; model c) and model d) have the same third state variable.



see that for a same state variable, its loading coefficients in different models (when it is constructed with different state variables) can be very different. Therefore the loading coefficients do not represent any generic nature of state variables.

3.1.2 A Better Measure of Contemporaneous Impact

Since the loading coefficients do not represent the generic nature (about the contemporaneous impact on the yield curve) of state variables, one wonders if there is a better measure of the contemporaneous impact that is inherent in state variables? In other words, can we find a measure of the contemporaneous impact that is invariant to different rotations? One way to proceed is to model the state variables such that their changes are independent of each other, and of zero unconditional mean. In this case, the scenario (a shock happens only in one state, others keep constant) in which the impact on the yield curve is represented by the loading coefficient is the same as the unconditional mean of all scenarios. Therefore the loading coefficients will be invariant as long as the state variables satisfy the above-mentioned conditions. However, this way is very inconvenient: in order to find the generic contemporaneous impact of three state variables, we might need to have three different realizations in each one of those only one state variable is of concern and others has no other use than identifying the contemporaneous impact of the concerned state.

In this section, I define a consistent and generic measure of the contemporaneous impact which is invariant to different rotations. The underlying idea is inspired by the generalized impulse response analysis developed in Koop et al. (1996) and Pesaran and Shin (1998): instead of assuming the scenario where only one state variable changes while others keep constant; we can assume when a shock happens in one state, there are also shocks happen in others, but we integrate out the effects of other shocks using the estimated distribution of the shocks.

Definition 1. Under a realization, given a time interval Δt, the *conditional contemporaneous impact* (CCI) of a vector of shocks δ happening in $\mathbf{i}$th state variable(s) $Z_{\mathbf{i},t}$ on the x-yr zero yield is the difference between the conditional expectations of the yield in Δt with the shocks and without the shocks:

$$\mathrm{CCI}_t\left(\delta,x,\Delta t,\mathbf{i}\right) \equiv \mathbb{E}_t^{\mathbb{P}}\left(y_{t+\Delta t}\left(x\right) | \Delta Z_{\mathbf{i},t+\Delta t} = \delta\right) - \mathbb{E}_t^{\mathbb{P}}\left(y_{t+\Delta t}\left(x\right)\right);$$

and the *unconditional contemporaneous impact* (UCI) is:

$$\mathrm{UCI}\left(\delta,x,\Delta t,\mathbf{i}\right) \equiv \mathbb{E}^{\mathbb{P}}\left(y_{t+\Delta t}\left(x\right) | \Delta Z_{\mathbf{i},t+\Delta t} = \delta\right) - \mathbb{E}^{\mathbb{P}}\left(y_{t+\Delta t}\left(x\right)\right),$$

where δ is $k\times 1$; the index vector $\mathbf{i}$ is also $k\times 1$ and a subset of $\{1,2,\cdots,m\}$; m is the number of factors in the model; $1\leq k\leq m$; $\Delta Z_{\mathbf{i},t}$ is the $\mathbf{i}$th elements of $\Delta Z_{t+\Delta t} = Z_{t+\Delta t} - Z_t$.

By their definitions, CCI and UCI are given by

when $k = m$

$$\begin{aligned}
\mathrm{CCI}_t\left(\delta,x,\Delta t,\mathbf{i}\right) &= \frac{\int_0^x \mathbf{C}\left(s\right)ds}{x}\left[\delta + \left(I - \exp\left(\mathbf{A}^{\mathbb{P}}\Delta t\right)\right)\left(Z_t - \mu\right)\right]; \\
\mathrm{UCI}\left(\delta,x,\Delta t,\mathbf{i}\right) &= \frac{\int_0^x \mathbf{C}\left(s\right)ds}{x}\delta;
\end{aligned}$$

when $1 \leq k < m$

$$\begin{aligned}
\mathrm{CCI}_t(\delta, x, \Delta t, \mathbf{i}) &= \frac{\int_0^x \mathbf{C}(s)\, ds}{x} \times \\
&\left[M^{\mathrm{p}}(\mathbf{i})^{-1} \begin{bmatrix} \delta \\ \Lambda_{\mathbf{2}} + \Omega_{\mathbf{2},\mathbf{1}} \Omega_{\mathbf{1},\mathbf{1}}^{-1} (\delta - \Lambda_{\mathbf{1}}) \end{bmatrix} + \left(I - \exp\left(\mathbf{A}^{\mathbb{P}} \Delta t \right) \right) (Z_t - \mu) \right] \\
\mathrm{UCI}(\delta, x, \Delta t, \mathbf{i}) &= \frac{\int_0^x \mathbf{C}(s)\, ds}{x} M^{\mathrm{p}}(\mathbf{i})^{-1} \begin{bmatrix} \delta \\ \Omega_{\mathbf{2},\mathbf{1}} \Omega_{\mathbf{1},\mathbf{1}}^{-1} \delta \end{bmatrix}.
\end{aligned}$$

The derivation and notation definitions are laid out in Appendix A. The alternative designed to replace the loading coefficients is UCI. Actually, the loading coefficient of, for example, the first state variable

$$\frac{\int_0^x \mathbf{C}(s)\, ds}{x} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

is given by

$$\mathrm{UCI}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, x, \Delta t, \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \right).$$

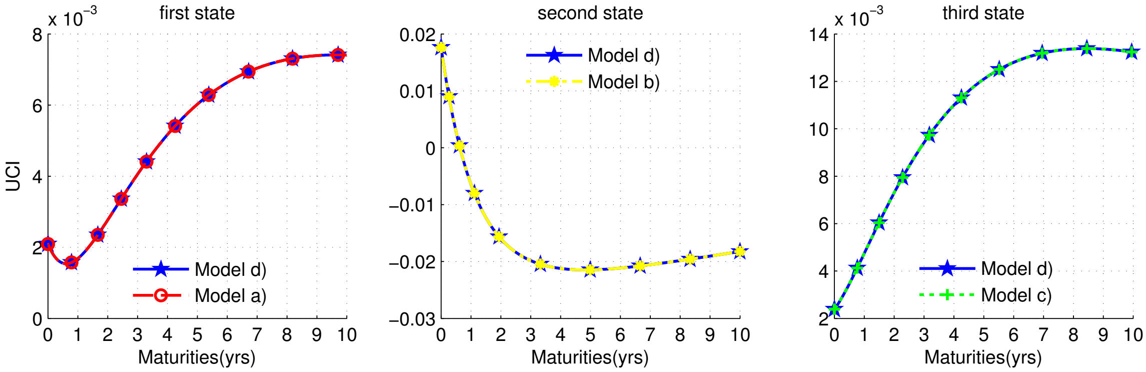
Therefore the loading coefficients essentially measure the impact of a special combination of shocks instead of the impact of a shock in one state variable.

Given the formula and estimates, it is ready to show the invariance of UCI graphically. To be comparable with the loading coefficients, I plot[2] UCI $\left(1, x, \frac{1}{252}, 1\right)$ for the first state variable in model d) and model a) (see Section 3.1.1 for the three models a, b, and c); UCI $\left(1, x, \frac{1}{252}, 2\right)$ for the second one in model d) and model b); and UCI $\left(1, x, \frac{1}{252}, 3\right)$ for the third one in model d) and model c). The results are shown in Figure 3.3. As expected, the UCIs of the three state variables are exactly the same in different models (the two curves plotted in each panel of Figure 3.3 are too close to each other to be distinguished). Therefore the UCI captures the inherent impact of the state variables on the yield curve and is invariant to rotations.

[2]Here I set $\Delta t = \frac{1}{252}$ (one day), which is the highest frequency of the data, to be make the impact the most "contemporaneous".

Figure 3.3: UCI in Different Models

This figure plots the UCI(from 1 day to 10 years) of three state variables (UCI $\left(1,x,\frac{1}{252},1\right)$, UCI $\left(1,x,\frac{1}{252},2\right)$, and UCI $\left(1,x,\frac{1}{252},2\right)$) in two different models.



3.2 Persistence Impact

The UCI and CCI presented in the last section provides us good measures on how changes in the state variables immediately impact the yield curve. These would be very useful for practitioners, as a good understanding on how yield curve movements follow the changes in the state variables can be the key to a successful trading strategy. However, it is also of a great concern to measure the persistence effect of shocks on the yield curve. Especially from a policy maker's perspective, one would love to know what is the expected long-term effect of certain shocks (either endogenous or exogenous) on the dynamic path of the interest rates. When policy makers are armed with this knowledge, the short interest rate as an instrument can be better deployed to stimulate or restrain the economy. In this section, in line with CCI and UCI, I define *conditional persistent impact* (CPI) and *unconditional persistent impact* (UPI) for analyzing the impact of state variables in terms of their persistent effects.

Definition 2. Under a realization, given a shock time interval Δt and a persistent time interval n, the *conditional persistent impact* (CPI) of a vector of shocks δ happening during Δt in $\mathbf{i}$th state variable(s) $Z_{\mathbf{i},t}$ on the x-yr zero yield is the difference between the conditional expectations of the yield in n with the shocks and without the shocks:

$$\mathrm{CPI}_t\left(\delta,x,\Delta t,n,\mathbf{i}\right)\equiv\mathbb{E}_t^{\mathbb{P}}\left(y_{t+n}\left(x\right)\middle|\Delta Z_{\mathbf{i},t+\Delta t}=\delta\right)-\mathbb{E}_t^{\mathbb{P}}\left(y_{t+n}\left(x\right)\right);$$

and the *unconditional persistent impact* (UPI) is:

$$\mathrm{UPI}(\delta,x,\Delta t,n,\mathbf{i}) \equiv \mathbb{E}^{\mathbb{P}}\left(y_{t+n}(x)\,|\,\Delta Z_{\mathbf{i},t+\Delta t}=\delta\right)-\mathbb{E}^{\mathbb{P}}\left(y_{t+n}(x)\right),$$

where δ is $k\times 1$; the index vector $\mathbf{i}$ is also $k\times 1$ and a subset of $\{1,2,\cdots,m\}$; m is the number of factors in the model; $1\leq k\leq m$; $\Delta Z_{\mathbf{i},t}$ is the $\mathbf{i}$th elements of $\Delta Z_{t+\Delta t}=Z_{t+\Delta t}-Z_t$.

By their definitions, CPI and UPI are given by

when $k=m$

$$\begin{aligned}\mathrm{CPI}_t(\delta,x,\Delta t,n,\mathbf{i}) &= \frac{\int_0^x \mathbf{C}(s)\,ds}{x}\exp\left(\mathbf{A}^{\mathbb{P}}(n-\Delta t)\right)\times \\ &\quad \left[\delta+\left(I-\exp\left(\mathbf{A}^{\mathbb{P}}\Delta t\right)\right)(Z_t-\mu)\right]\end{aligned}$$

$$\mathrm{UPI}(\delta,x,\Delta t,n,\mathbf{i}) = \frac{\int_0^x \mathbf{C}(s)\,ds}{x}\exp\left(\mathbf{A}^{\mathbb{P}}(n-\Delta t)\right)\delta.$$

when $1\leq k<m$

$$\begin{aligned}\mathrm{CPI}_t(\delta,x,\Delta t,\mathbf{i}) &= \frac{\int_0^x \mathbf{C}(s)\,ds}{x}\exp\left(\mathbf{A}^{\mathbb{P}}(n-\Delta t)\right)\times \\ &\quad \left[M^{\mathrm{p}}(\mathbf{i})^{-1}\begin{bmatrix}\delta \\ \Lambda_{\mathbf{2}}+\Omega_{\mathbf{2},\mathbf{1}}\Omega_{\mathbf{1},\mathbf{1}}^{-1}(\delta-\Lambda_{\mathbf{1}})\end{bmatrix}+\left(I-\exp\left(\mathbf{A}^{\mathbb{P}}\Delta t\right)\right)(Z_t-\mu)\right] \\ \mathrm{UPI}(\delta,x,\Delta t,\mathbf{i}) &= \frac{\int_0^x \mathbf{C}(s)\,ds}{x}\exp\left(\mathbf{A}^{\mathbb{P}}(n-\Delta t)\right)M^{\mathrm{p}}(\mathbf{i})^{-1}\begin{bmatrix}\delta \\ \Omega_{\mathbf{2},\mathbf{1}}\Omega_{\mathbf{1},\mathbf{1}}^{-1}\delta\end{bmatrix}.\end{aligned}$$

The derivation and notation definitions are again given in Appendix A.

4 Calibrating the $\mathbb{P}$-measure parameters

4.1 Predicting future changes using state variables

The following regression is run for each maturity x and horizon h from one week to five years with an increment of one week:

$$\Delta y_{t+h,x} = \alpha_h + \beta_h^{\intercal} Z_t + \varepsilon_t \tag{4.1}$$

where $\Delta y_{t+h,x} = y_{t+h}(x) - y_t(x)$. Given the $\mathbb{P}$-measure dynamic of Z_t in (2.2), $\Delta y_{t+h,x}$ is given by

$$\Delta y_{t+h,x} \quad = \quad \frac{\int_0^x \mathbf{C}_{\text{Base}}(s)\, ds}{x} \left(I - \exp\left(\mathbf{A}^{\mathbb{P}}_{\text{Base}}\right)\right)\left(\mu_{\text{Base}} - Z_t\right) + \xi_t, \tag{4.2}$$

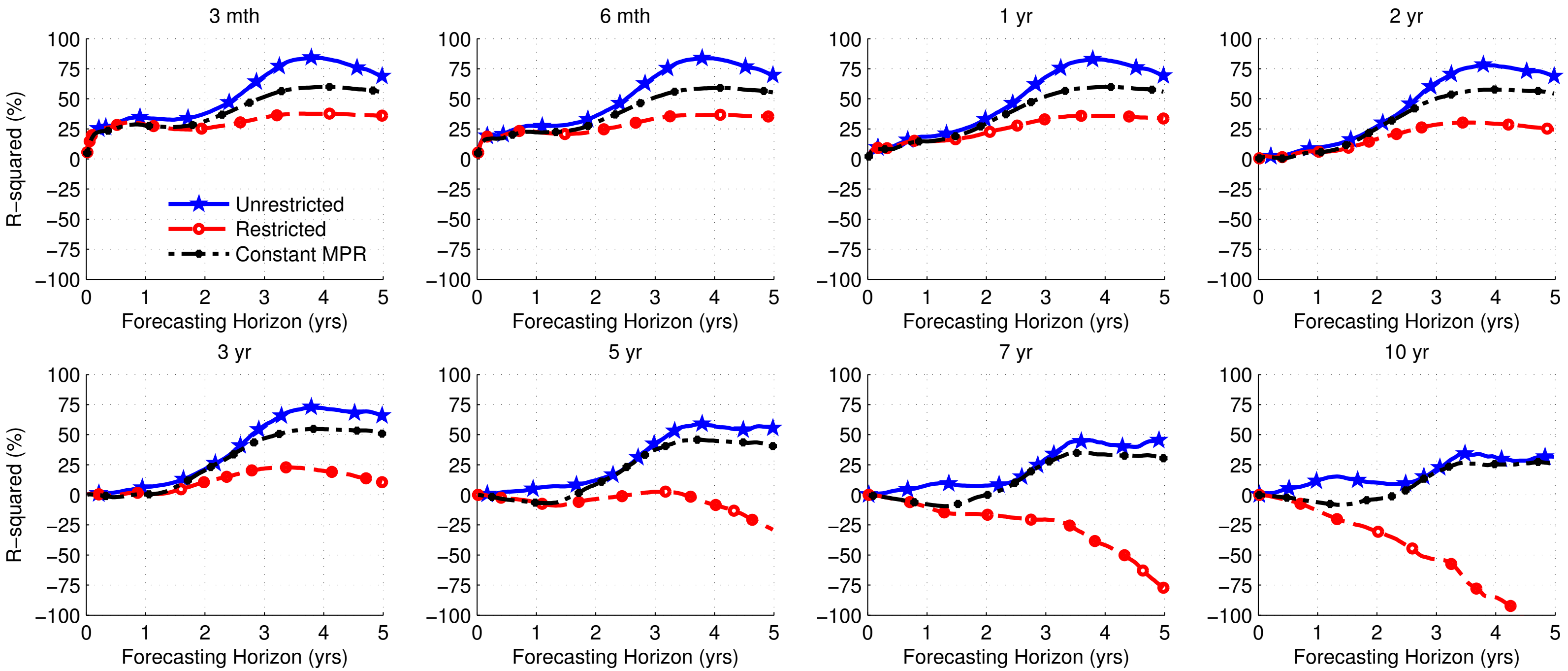
where $\xi_t = \frac{\int_0^x \mathbf{C}_{\text{Base}}(s)dx}{x} \int_0^h \exp\left(\mathbf{A}^{\mathbb{P}}_{\text{Base}}(h-s)\right) \mathbf{B}_{\text{Base}} dW^{\mathbb{P}}_s$. If we assume constant market price of risks (MPR), then we have

$$\Delta y_{t+h,x} \quad = \quad \frac{\int_0^x \mathbf{C}_{\text{Base}}(s)\, ds}{x} \left(I - \exp\left(\mathbf{A}_{\text{Base}}\right)\right)\left(\mu'_{\text{Base}} - Z_t\right) + \xi'_t \tag{4.3}$$

where $\xi'_t = \frac{\int_0^x \mathbf{C}_{\text{Base}}(s)ds}{x} \int_0^h \exp\left(\mathbf{A}_{\text{Base}}(h-s)\right) \mathbf{B}_{\text{Base}} dW^{\mathbb{P}}_s$. To see how well the state variables predict (in sample prediction) the future changes of the yields, I plot R^2s of the three equations above in Figure 4.1. From the plots we can see that (4.1) always gives the highest R^2s, and these R^2 curves peak around 3.8 years for all maturities; R^2s from (4.3) are constantly lower than those from (4.1), and the difference is larger for shorter maturities (3-mth to 3-yr) at longer horizons (3 to 5yrs) and longer maturities (5- to 10-yr) at shorter horizons (less than 2yrs), however, they still show significant predicting power; R^2s from (4.2) are always the lowest, and they get worse when maturity gets longer: for example, the RHS of (4.2) shows no predicting power at all horizons for maturities longer than 5-yr.

Figure 4.1: R^2s from regressing future changes of the yields on state variables

This figure shows the R^2s from (4.1) to (4.3). R^2s are computed for (4.1) by running OLS; R^2s are computed for (4.2) by setting all parameters be their MLE estimates; R^2s are computed for (4.3) by setting all parameters except μ'_{Base} be their MLE estimates, and μ'_{Base} is estimated by running OLS while fixing other parameters. R^2s from (4.1), (4.2), and (4.3) are labeled by "Unrestricted", "Restricted", and "Constant MPR" respectively. The changes are measured at horizons from one week to five years.



First of all, the results again confirm the bias of the ML estimates of the $\mathbb{P}$-measure parameters mentioned in Section 2.3, as they completely fail to capture the predicting power implicit in the state variables. Secondly, the R^2 curve of (4.3) essentially represents the predicting power of the yield curve slope, which is the risk neutral expectation of the future change of short rate; and this is a sum of the market expectation and the term premium. It is well known that the term premium is not constant over time, therefore the R^2 curve of (4.3) should be considered as a soft lower bound of the R^2s of the market expectations, in other words, the R^2s of the market expectations should be higher than those of (4.3) in an average sense. Thirdly, it is found that when the state variables are freed (see (4.1)) from their estimated $\mathbb{P}$ or $\mathbb{Q}$ dynamics, they show strong predicting power, especially when the forecasting horizon is around 3-4yrs. The R^2 curve of (4.1) can be regarded as an upper bound of the R^2s of the market expectations. Then some naturally follow-up questions would be: is there a set of $\mathbb{P}$-measure parameters that is able to capture the R^2 curve of (4.1)? what can we tell about the market expectation from these results? I address these questions in the following subsection.

4.2 Unconstraint v.s. constraint

Unfortunately there does not exist a set of $\mathbb{P}$-measure parameters that is able to completely capture the observed R^2 curves.[3] However, we can find a set of $\mathbb{P}$-measure parameters from which the resulting R^2 curves best fit the observed ones in a least square sense. Specifically, the calibrated $\mathbb{P}$-measure parameters, $\mathbf{A}^{\mathbb{P}}_{\text{Base}}$ and μ_{Base}, are solutions to the following unconstraint optimization problem:

$$\underset{\{\mathbf{A}^{\mathbb{P}}_{\text{Base}},\mu_{\text{Base}}\}}{\arg\max} \left\{\text{average } R^2 \text{of (4.2) across } h' \text{and } m\right\}, \tag{4.4}$$

where $h' = \begin{bmatrix} \text{3mths}, & \text{1yr}, & \text{3.5yrs}, & \text{5yrs} \end{bmatrix}$. The resulting R^2 curves are shown in Figure 4.2.

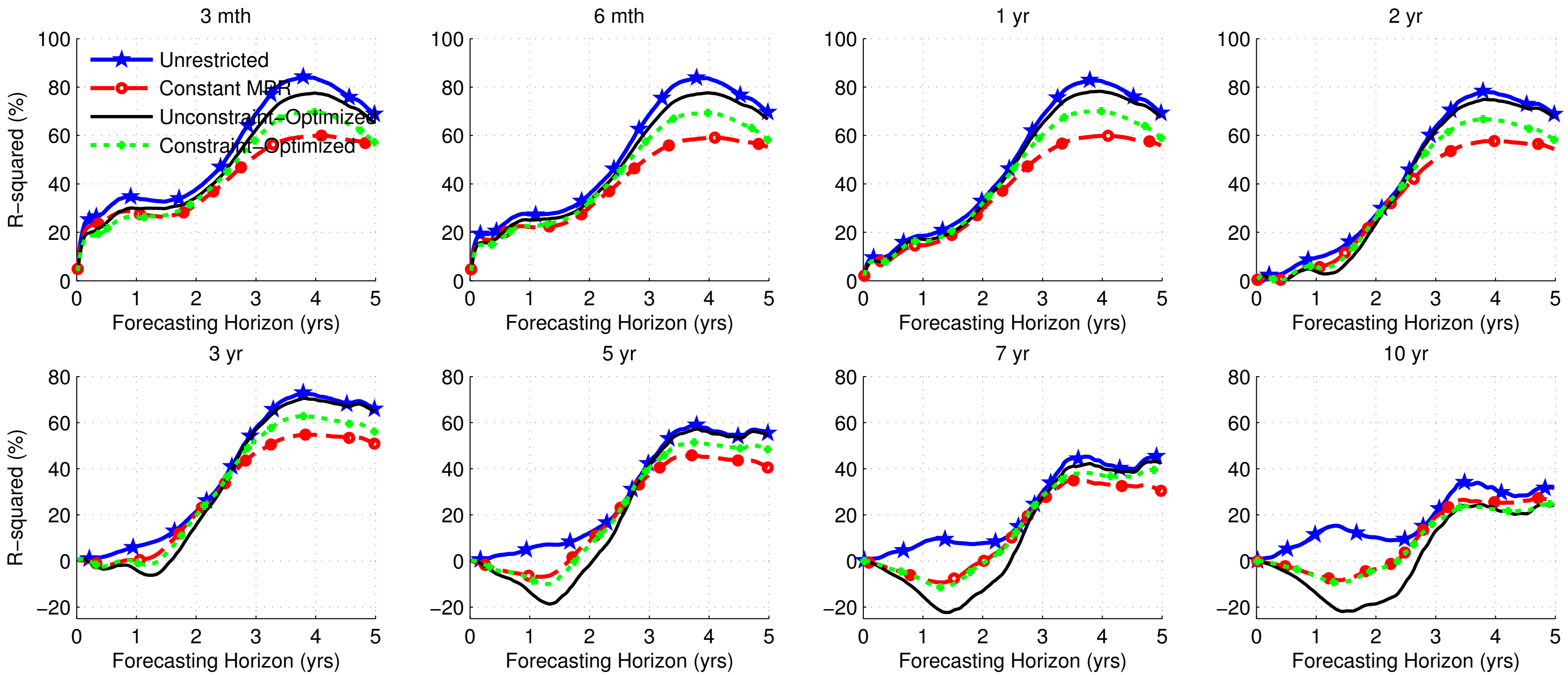
From the plots we can see that the unconstraint optimization results in much better fitting in shorter maturities than in longer maturities. It fits best at 1-yr maturity where the unconstraint optimized R^2 curve matches the unrestricted R^2 curve almost perfectly.

[3]This clearly indicates that the essentially affine market price of risk specification is not flexible enough to capture the empirical observations in this regard.

However, the fit deteriorates significantly as the maturity gets longer: from 2-yr to 7-yr the unconstraint optimization is even worse than the constant MPR at horizons from 1 week to 3 years, although it still fits the unrestricted curves best at horizons from 3 to 5 years; when the maturity comes to 10-yr, the unconstraint optimization becomes worst, it is outperformed by the constant MPR at all horizons from 1 week to 5 years. Apparently, the unconstraint optimization gives us an asymmetric predicting performance across different maturities and over different horizons. However, obtaining better fits is not the focus in the current paper. Here I care more about how legitimate the resulting market expectation and term premium are. In order to better understand the question, let's first look at the definitions of the market expectation and term premium, and how they are linked with yield (forward) curve.

Figure 4.2: R^2 curves comparison

This figure compares the R^2 curves from (4.1) and (4.3) with those resulting from unconstraint optimization (4.4) and constraint optimization (4.5). R^2s are computed for (4.1) by running OLS; R^2s are computed for (4.3) by setting all parameters except μ'_{Base} be their MLE estimates, and μ'_{Base} is estimated by running OLS while fixing other parameters; R^2s from unconstraint (constraint) optimization are computed by plugging the calibrated $\mathbb{P}$-measure parameters of unconstraint (constraint) optimization into (4.2). R^2s from (4.1), (4.3), (4.4) and (4.5) are labeled by "Unrestricted", "Constant MPR", "Unconstraint-Optimized", "Constraint-Optimized" respectively. The changes are measured at horizons from one week to five years.



It is trivial to see that the instantaneous forward rate at maturity x, $r(t,x)$ is a sum of a constant, short rate, market expected change of the short rate, and instantaneous forward term premium:

$$\begin{aligned} r(t,x) &= \underbrace{\Theta^*(h) - \Theta^*(0)}_{\text{constant}} + \underbrace{r(t,0)}_{\text{short rate}} \\ &+ \underbrace{\left[\mathbb{E}_t^{\mathbb{P}}(r(t+x,0)) - r(t,0)\right]}_{\text{market expected change of short rate}} + \underbrace{\left[\mathbb{E}_t^{\mathbb{Q}}(\Delta r(t+x,0)) - \mathbb{E}_t^{\mathbb{P}}(\Delta r(t+x,0))\right]}_{\text{instantaneous forward term premium}}. \end{aligned}$$

Therefore, averagely speaking a positive shock in any of these three variables should give $r(t,x)$ a positive contemporaneous shock. Now, let's check if the market expectation and term premium, which are constructed from unconstraint optimized $\mathbb{P}$-measure parameters, and the short rate impact the instantaneous forward curve in the way as predicted.

To see their unconditional contemporaneous impact on the forward rates, first of all, we need to incorporate these three variables in a realization. Towards this end, I define a x-specific transfer matrix

$$M(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \quad 0 \quad 0 \\ \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \left(\exp\left(M^0 \mathbf{A}_{\text{Base}}^{\mathbb{P}} \left(M^0\right)^{-1} x \right) - I \right) \\ \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \left(\exp\left(M^0 \mathbf{A}_{\text{Base}} \left(M^0\right)^{-1} x \right) - I \right) \end{bmatrix} M^0,$$

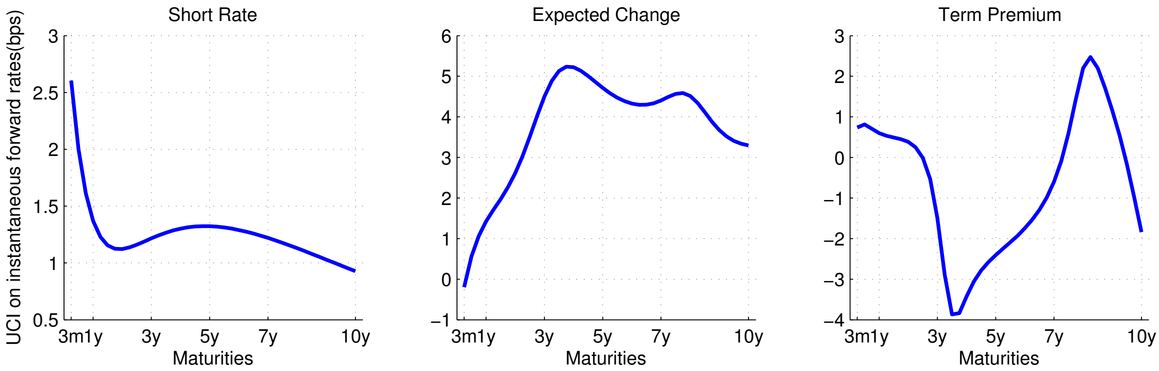
where

$$M^0 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The three states in $Z_t(x) = M(x) Z_t^{\text{Base}}$ represent the short rate, the expected change of the short rate in x yrs, and the corresponding instantaneous term premium respectively. I denote this realization by "x-forward term premium" realization. As here the focus is instantaneous forward curve instead of zero yield curve, I use UCI^{fwd} which is a variant of UCI defined in Section 3.1.2 to measure the unconditional contemporaneous impact,

Figure 4.3: $\text{UCI}^{\text{fwd}}_{M(x)}$ in the unconstraint optimization

This figure plots the $\text{UCI}^{\text{fwd}}_{M(x)}$ (from 3-mth to 10-yr) of one standard deviation of the daily change of each state variable in different "x-forward term premium" realizations constructed using $\mathbb{P}$-measure parameters from the unconstraint optimization.



and it is given by

$$\text{UCI}^{\text{fwd}}_{M(x)}(\delta, x, \Delta t, \mathbf{i}) = \mathbf{C}(x) M^{\text{p}}(\mathbf{i})^{-1} \begin{bmatrix} \delta \\ \Omega_{\mathbf{2},\mathbf{1}} \Omega^{-1}_{\mathbf{1},\mathbf{1}} \delta \end{bmatrix},$$

where the subscript of $M(x)$ indicates that $\text{UCI}^{\text{fwd}}_{M(x)}$ is defined under the "x-forward term premium" realization. Now, I let x run from 3-mth to 10-yr, and plot $\text{UCI}^{\text{fwd}}_{M(x)}$ of a shock of one standard deviation of the daily change in each state variable: $\Delta Z_t(x) = Z_t(x) - Z_{t-\frac{1}{252}}(x)$ against x in Figure 4.3.

Using the calibrated $\mathbb{P}$-measure parameters from the unconstraint optimization, the short rate and expected change of the short rate have the expected UCI on the forward curve, however, the term premium fail to have the expected UCI. As shown in the figure, at most of the maturities up to 10-yr, the term premium's UCI is negative. This means that for a maturity x, when the corresponding term premium increases, the instantaneous forward rate decreases. This observation is counter-intuitive for a well-defined term premium. Since the expected change of the short rate has positive UCI at most of the maturities, the negative UCI of the term premium indicates a strong negative correlation between the expected change and term premium, meaning when the term premium increases the expected change decreases even more such that the net effect on the forward curve is negative. Indeed, the average correlation coefficient between the expected change and the term premium across all maturities is -34.3%. Therefore, the term premium constructed using the unconstraint optimized $\mathbb{P}$-measure parameters is not well-defined; equivalently, the resulting expected change of the short rate is not a legitimate measure of the market expectations.

Given this insight, I conduct a constraint optimization to calibrate a new set of $\mathbb{P}$-measure parameters which are solutions to the following problem:

$$\underset{\{\mathbf{A}^{\mathbb{P}}_{\text{Base}},\mu_{\text{Base}}\}}{\arg\max}\ \left\{\text{average } R^2 \text{of (4.2) across } h' \text{and } m\right\}, \tag{4.5}$$

s.t.

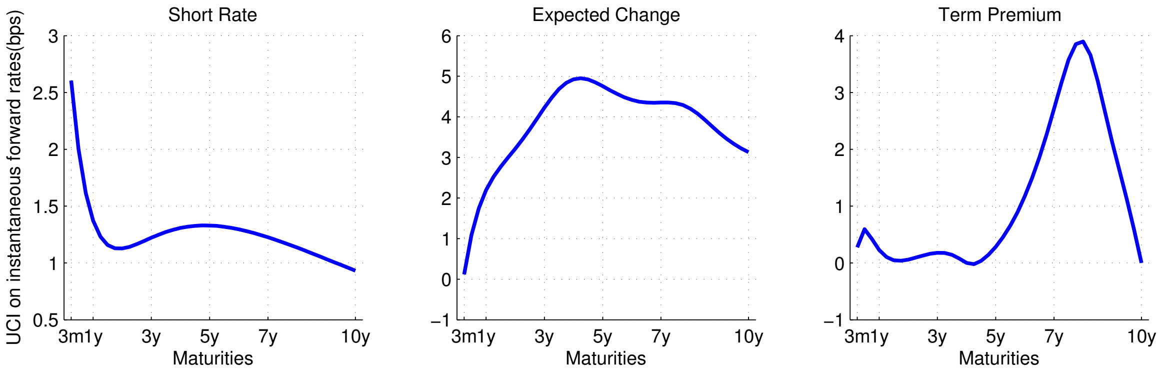
$$\text{UCI}^{\text{fwd}}_{M(x)}(1,x,\Delta t,2) > 0\,\&\,\text{UCI}^{\text{fwd}}_{M(x)}(1,x,\Delta t,3) > 0$$

where $x = \begin{bmatrix} \text{3-mth}, & \text{4-yr}, & \text{10-yr} \end{bmatrix}$, $\Delta t = 1/252$. The the fitted R^2 curves and $\text{UCI}^{\text{fwd}}_{M(x)}$ of the constraint optimization are shown in Figure 4.2 and Figure 4.4 respectively. Regarding the fitted R^2 curves, the constraint optimization outperforms the constant MPR in most of the cases, this is consistent with the statement I make in Section 4.1 that the R^2 curves of the constant MPR should be regarded as the lower bounds of the R^2s of the market expectations about the future changes of interest rates. It is also interesting to see that the constraint optimization outperforms more at shorter maturities and longer horizons, then it gets closer to the lower bound when the maturity gets longer. This just reconciles with the factor that the term premium of a longer maturity zero yield, which is the average of the forward term premia, is more constant than that of a shorter maturity zero yield. So the constraint optimization gives us a more symmetric predicting performance across different maturities and over different horizons. Of course as shown in Figure 4.4 the $\text{UCI}^{\text{fwd}}_{M(x)}$s in the constraint optimization have the correct signs by construction. Noticed that the average correlation coefficient between the expected change and the term premium across all maturities is now 2.6%, remembering that the constraints are not (directly) imposed on the correlation coefficient.

Given above empirical evidence, I therefore assume the constraint optimization gives us a set of $\mathbb{P}$-measure parameters that constructs more legitimate measures of the market expectations and term premium. Therefore the $\hat{\mathbf{A}}^{\mathbb{P}}_{\text{Cali,Base}}$, the calibrated estimate of $\mathbf{A}^{\mathbb{P}}_{\text{Base}}$ by the constraint optimization will be used in the following analysis.

Figure 4.4: $\text{UCI}^{\text{fwd}}_{M(x)}$ in the constraint optimization

This figure plots the $\text{UCI}^{\text{fwd}}_{M(x)}$ (from 3-mth to 10-yr) of one standard deviation of the daily change of each state variable in different "x-forward term premium" realizations constructed using $\mathbb{P}$-measure parameters from the constraint optimization.



5 State Variables

5.1 Short rate

Notice that

$$\mathbf{C}(0)_{\text{Base}} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}.$$

Therefore if we define an M^0 as

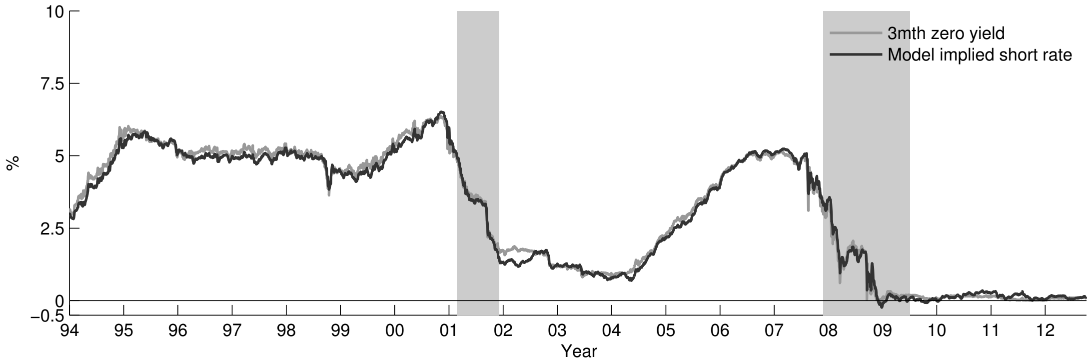
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

then the first state in a new set of state variables $Z_t^0 = M^0 Z_t^{\text{Base}}$ will be precisely the short rate plus a constant:

$$\begin{aligned} r(t,0) &= \varphi + \frac{1}{2}\mathbf{C}(0)_{\text{Base}}\left(\mathbf{A}_{\text{Base}}^{-1}\mathbf{B}_{\text{Base}}\mathbf{B}_{\text{Base}}^{\intercal}\left(\mathbf{A}_{\text{Base}}^{\intercal}\right)^{-1}\right)\mathbf{C}(0)_{\text{Base}}^{\intercal} + \mathbf{C}(0)_{\text{Base}} Z_t^{\text{Base}}, \\ &= \varphi + \frac{1}{2}\mathbf{C}(0)_{\text{Base}}\left(\mathbf{A}_{\text{Base}}^{-1}\mathbf{B}_{\text{Base}}\mathbf{B}_{\text{Base}}^{\intercal}\left(\mathbf{A}_{\text{Base}}^{\intercal}\right)^{-1}\right)\mathbf{C}(0)_{\text{Base}}^{\intercal} + \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\left(M^0\right)^{-1} Z_t^0 \\ &= \varphi + \frac{1}{2}\mathbf{C}(0)_{\text{Base}}\left(\mathbf{A}_{\text{Base}}^{-1}\mathbf{B}_{\text{Base}}\mathbf{B}_{\text{Base}}^{\intercal}\left(\mathbf{A}_{\text{Base}}^{\intercal}\right)^{-1}\right)\mathbf{C}(0)_{\text{Base}}^{\intercal} + \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} Z_t^0. \end{aligned}$$

Figure 5.1: Model Implied Short Rate v.s. 3-mth Zero Yield

This figure compares the dynamics of the model implied short rate and 3mth zero yield from 1994 to 2012.



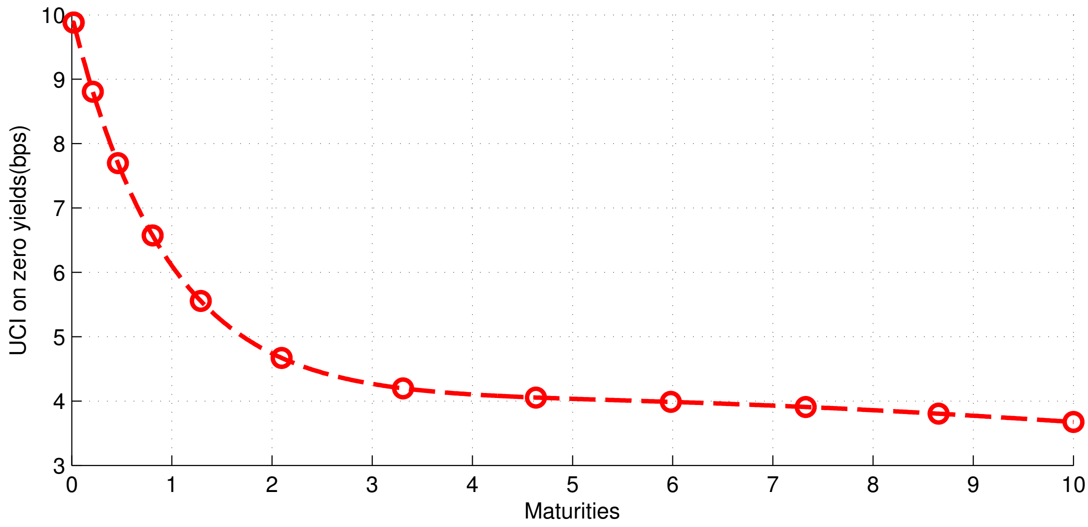
To confirm that the first state variable in the Z_t^0 represents the short rate, I compare the model implied short rate, which is $Z_{1,t}^0$ (the first state in Z_t^0) plus a constant, with the shortest yield, 3-mth zero yield, used in the estimation. Although the short rate is not the 3-mth zero yield as the short rate is with 0 time-to-maturity, they should look very alike since their maturities are close. The comparison is shown in Figure 5.1. By eyeballing, the model implied short rate is very close to the 3-mth zero yield. Their correlation is as high as 99.7%. In Piazzesi (2005), the correlation between the short rate implied by her model and LIBOR rate is only 54%.

Figure 5.2 shows the UCI of the short rate. From the figure, first of all we can see the impact of the short rate quickly losses 60% its power at maturities from 0 to 3-year. Then the remaining 40% almost flatly lands on maturities of 3 to 10-year. This means a significant portion (40%) of the short rate shock has a uniform impact on the middle part and the long end of the yield curve. Specifically, in an average sense, 10bps increase in the short rate can immediately induce about 4bps increase in 4- to 10-year zero yields.

Figure 5.3 shows the PCI of the short rate over 4 years of time. From Figure 5.2, we see that 40% of the impact of a shock in short rate lands on the longer maturity zero yields. Now we can also see that this portion of the impact on the long maturity yields is quite persistent (at least over 4 years of time). This observation validates Fed's conventional monetary actions in which the Fed tries to affect the long term yields by

Figure 5.2: UCI of the short rate against maturities

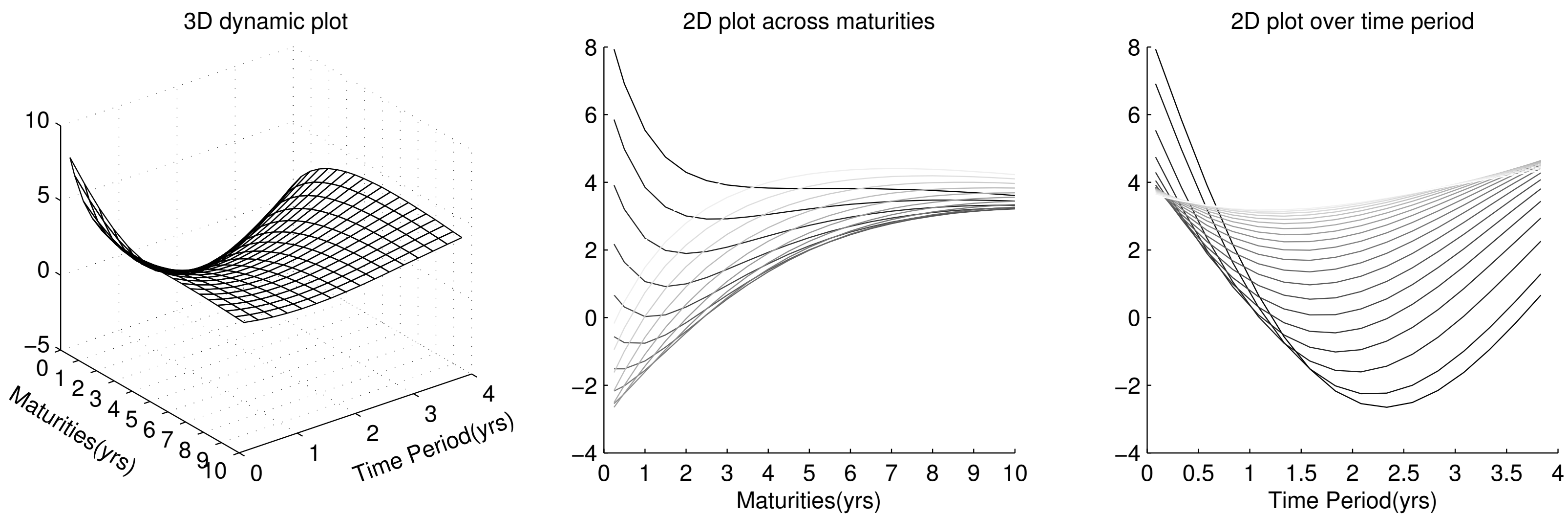
This figure plots UCI of the short rate given a shock of ten bps increase in the short rate in a day against maturities from 1-week to 10-year.



manipulating the short rate. While the impact on the long term yields is persistent, the impact on the short term yields is transient: the impact on the yields with maturities shorter than 3-yr decays quickly to zero within 1 year of time, and it even becomes negative after one year. For a 10bps shock in the short rate, the effect on short term zero yields becomes zero in around one year, and reaches the bottom value of -2.5bps in around 2 years, then returns to zero in 4 years. This observation shows that the short end of the yield curve presents resilience when absorbing shocks. This is important for trend following bond traders to specify the trading timing.

Figure 5.3: UPI of the short rate against maturities and horizons

This figure shows how the zero yield curve responds to a shock of ten bps one day increase in the short rate over a 4 yrs of time. (the left panel is 3D dynamic plot showing how the impact on the yield curve up to 10-yr maturity envolvs with n changes from 0 to 4; the middel panel is a 2D version of the left panel in which the darker(lighter) curve represents the impact on the whole yield curve when n is smaller(larger); the right panel is another 2D version of the left panel in which the darker(lighter) curve shows how the impact on the shorter(longer) maturity yield changes over the 4 yrs of time.



5.2 Market expectations and term premia

The fact that the R^2 curves peak around 3.5 years means the market expectation best predicts the future yield curve at this horizon. Therefore, in this section $h = 3.5$ is used for the purpose of illustrating the model-implied market expectations and term premia.

Here I define the *market expectation* state variable as the average expected future short rate over horizon h; and the *term premium* state variable is the difference between h-yr zero yield and the market expectation state variable. Specifically, the relation between the h-yr zero yield, the average expected future short rate, and yield term premium is described by the following equation

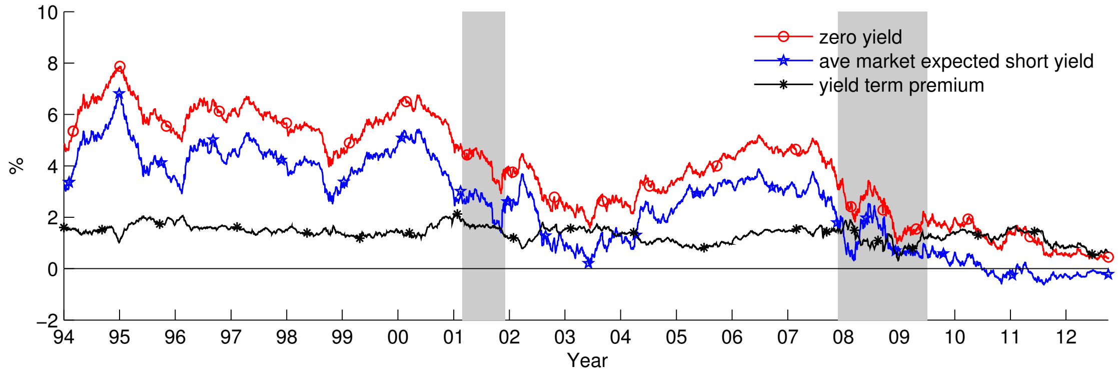
$$\text{ytp}_t^h = y_t(h) - \text{aesr}_t^h, \tag{5.1}$$

where "ytp" and "aesr" stand for the yield term premium and the average expected short rate, respectively; $\text{aesr}_t^h \equiv \frac{\int_0^h \mathbb{E}_t^{\mathbb{P}}[r(t+s,0)]ds}{h}$.

Figure 5.4 shows the the dynamics of the 3.5-yr zero yield, the market expectation, and term premium. Two observations are worth noting: a) in the period of the conundrum during 2004-05, it is well documented (see, e.g., Kim and Wright, 2005; Bernanke, 2006; Cochrane and Piazzesi, 2008; Joslin et al., 2010) that there was a drop in term premia at long maturities. Here I show that there also was a drop in term premia in the short end although the short maturity yield was increasing during that period. It is believed that the net demand for long-term issues had increased. This lowered the long end of the yield curve while lifting the short end. However, more stable inflation, better-anchored inflation expectations, and a reduction in economic volatility did reduce the market uncertainty. Therefore even the short maturity term premia also dropped; b) after late of 2010 the market expected short rate became negative, therefore the short maturity yield mainly consisted of the term premia. However, the term premium has been shrinking since mid of 2011 as the market expectation has been increasing and short maturity yield kept decreasing.

Figure 5.4: 3.5-yr Zero yield, average market expected short rate, and yield term premium

This figure plots the dynamics of the 3.5-yr zero yield, the average market expected short rate, and the yield term premium from 1994 to 2012.



6 Case Studies: impacts of LSAP, MEP, and QE3 announcements

In December 2008, the Federal Reserve reduced its target for the federal funds rate—the traditional tool of U.S. monetary policy—to a range of 0 to 25 basis points, essentially the lower bound of zero. This means that despite the severity of the recession, the conventional option of reducing the Funds rate was no longer available. Concerned that economic conditions would deteriorate, the FOMC chose to pursue unconventional monetary policies since there was no scope for further cuts in short-term interest rates.

On November 25, 2008, the Federal Reserve announced that it would purchase up to $100 billion in agency debt, and up to $500 billion in agency MBS, and later on a series of large-scale asset purchase programs (LSAP) were implemented on those debts and long-term Treasury securities as well. LSAP is also referred to as “Quantitative Easing” (QE). On September 21, 2011, the FOMC announced the Maturity Extension Program (MEP), under which the FOMC will purchase, by the end of June 2012, $400 billion of Treasury securities with remaining maturities of 6 to 30 years while simultaneously selling an equal amount of Treasuries with remaining maturities of 3 years or less. MEP is also referred to as “operation twist”. On September 12 and 13, 2012, the third round

Figure 6.1: Changes upon the LSAP announcement

This figure shows changes of the zero yield curve, aesr_t^h, and ytp_t^h happened on November 25, 2008 when the Fed annouced the LSAP. h runs from 0 to 10-yr.

of QE, “QE3" was announced , which entails buying \$40 billion in mortgage-backed securities per month. And no specific end date was mentioned.

In this section, I show how these three announcements affect aesr_t^h and ytp_t^h for different hs (see 5.1), and how these announcements persistently impact the yield curve in a sense of impulse response, using the *CPI* I defined in Section3.2. Since we already know the ΔZ_t on these announcement days, CPI with $k = m$ will be used to infer the persistent impact. Notice that when $k = m$, CPI is independent of state variable rotation, meaning there is no need to define it under a specific realization.[4]

For the LSAP and MEP, from the zero yield curve changes shown in the left panels of Figure 6.1 and Figure 6.2, I find that the contemporaneous responses of market were consistent with the intents of the Fed: the LSAP was targeting the long-term yields, in Figure 6.1 it shows that the zero yield curve was tilted clockwise with the short end being intact; in Figure 6.2 the zero yield curve rose in the short end but fell in the long end although not to a significant extent, this is a response to the “twist” effect of the MEP. I also find that the MEP had a significantly smaller impact on the zero yield curve, this is somewhat expected, as the MEP can actually be considered as a part of the LSAP.

However, for the QE3, from the left panel of Figure 6.3, it is found that the con-

[4] So are UPI, CCI, and UCI.

Figure 6.2: Changes upon the MEP announcement

This figure shows changes of the zero yield curve, aesr_t^h, and ytp_t^h happened on September 21, 2011 when the Fed annouced the MEP. h runs from 0 to 10-yr.

temporaneous responses of the zero yield curve did not go as what the Fed intended: although just as the LSAP, the QE3 was also targeting the long-term yields, in Figure 6.3 it shows that the zero yield curve is tilted counter-clockwise with the short end being a bit flatter. In other words, upon the announcement of the QE3, the long-term yields actually increased while the short-term yields decreased.

From the middle and right panels of Figure 6.1, I find that the market seemed (at least on the day of the LSAP announcement) to have a strong believe that the LSAP would have a positive influence on the recovery of the economy instead of just boosting up the inflation without stimulating the recovery: upon the LSAP announcement, the aesr_t^h fell significantly at most of maturities (the exceptions only happened around the maturity of 2-yr, and the extent is small), the longer maturity the larger extent; and the ytp_t^h fell at all maturities considered meaning the LSAP announcement reduced the market uncertainty by a significant amount.

Figure 6.3: Changes upon the QE3 announcement

This figure shows changes of the zero yield curve, aesr_t^h, and ytp_t^h happened on September 12 and 13, 2012 when the Fed annouced the QE3. h runs from 0 to 10-yr.

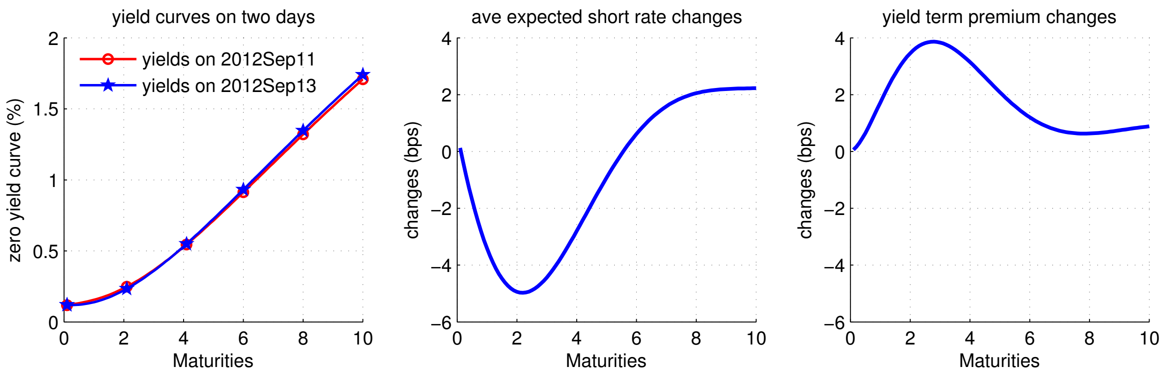


Figure 6.4: CPI of the LSAP announcement on the zero yield curve

This figure shows how the zero yield curve responds to the LSAP announcement on November 25, 2008 over a 4 yrs of time. (the left panel is 3D dynamic plot showing how the impact on the yield curve up to 10-yr maturity envolvs with n changes from 0 to 4; the middel panel is a 2D version of the left panel in which the darker(lighter) curve represents the impact on the whole yield curve when n is smaller(larger); the right panel is another 2D version of the left panel in which the darker(lighter) curve shows how the impact on the shorter(longer) maturity yield changes over the 4 yrs of time.

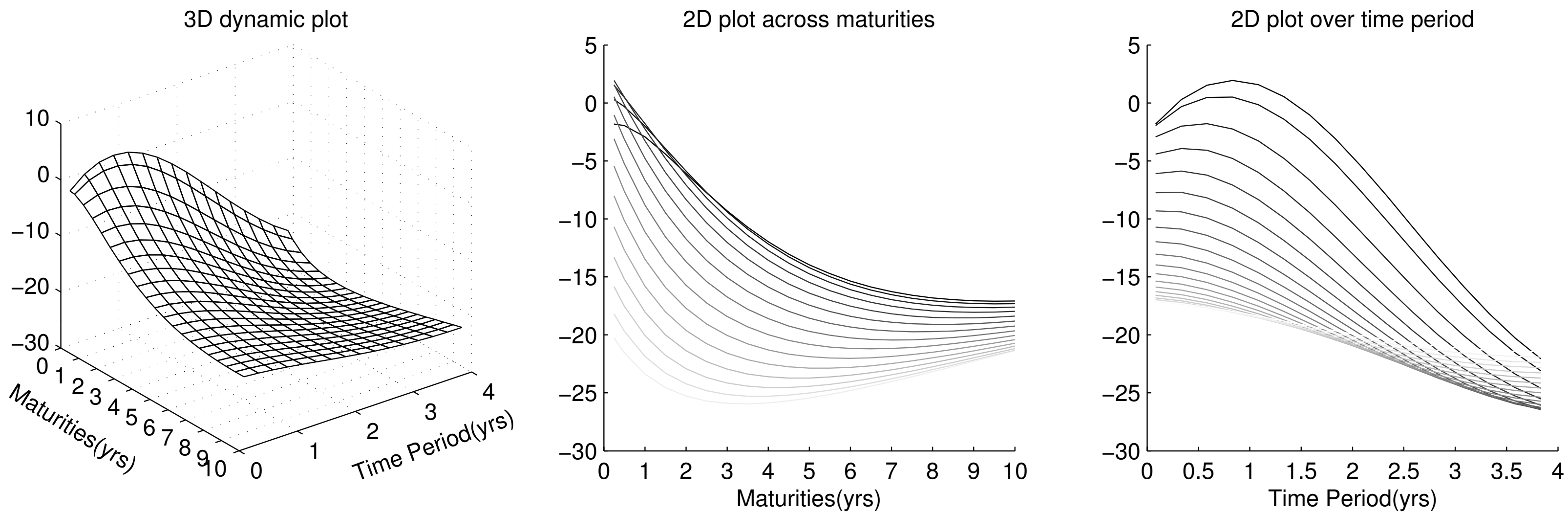


Figure 6.4 shows the expected impacts of the LSAP announcement on the yield curve 4 years into the future. It is found that the expected long run impact of the LSAP announcement is significant and persistent: the LSAP announcement had a persistent impact on the long end of the yield curve and a gradually increasing impact on the short end, specifically, the LSAP announcement lowered the long end by 16bps instantly and the impact would stay there for at least 4 years; although it only contemporaneously lowered the short end by about 3bps, the impact would be gradually increasing, it is expected to lower the short end by 20bps in 4 years of time.[5]

The MEP announcement had a much smaller impact on the yield curve, and it impacted the market expectations quite differently. As we can see in the middle panel of Figure 6.2, the announcement of MEP increased the aesr_t^h for $h < 6$-yr, and decreased aesr_t^h for $h > 6$-yr. This means that the market expected short-term yields to increase (or decrease less) while still expecting long-term yields to decrease (or increase less) by a small extent. This observation confirms that the MEP announcement controlled the market expectations successfully: upon the announcement the market believed that the additional purchase of $400 billion of long term Treasury securities financed by selling a same amount of the short term securities would increase the short term yields in the short run and lower the long term yields. And from the right panel of Figure 6.2, we can see the MEP announcement had a similar negative impact on ytp_t^h, but with smaller extents. This means that the MEP announcement also somewhat reduced the market uncertainty.

However, the expected long run impact of the MEP announcement was much less notable. This is confirmed in Figure 6.5, specifically, the announcement was expected to lower the yield curve by about 4bps in 4 years of time, which was only $\frac{1}{6}$ of the impact of the LSAP announcement. One observation worth mentioning is that, unlike the LSAP, the MEP announcement was expected to increase the short rate in about 3 years before it eventually pushing down the short rate. This is consistent with the increase of the market expectations at short maturities.

[5] Of course the downward impact on the short end would be bounded by the zero lower bound.

Figure 6.5: CPI of the MEP announcement on the zero yield curve

This figure shows how the zero yield curve responds to the MEP announcement on September 21, 2011 over a 4 yrs of time. (the left panel is 3D dynamic plot showing how the impact on the yield curve up to 10-yr maturity envolvs with n changes from 0 to 4; the middel panel is a 2D version of the left panel in which the darker(lighter) curve represents the impact on the whole yield curve when n is smaller(larger); the right panel is another 2D version of the left panel in which the darker(lighter) curve shows how the impact on the shorter(longer) maturity yield changes over the 4 yrs of time.

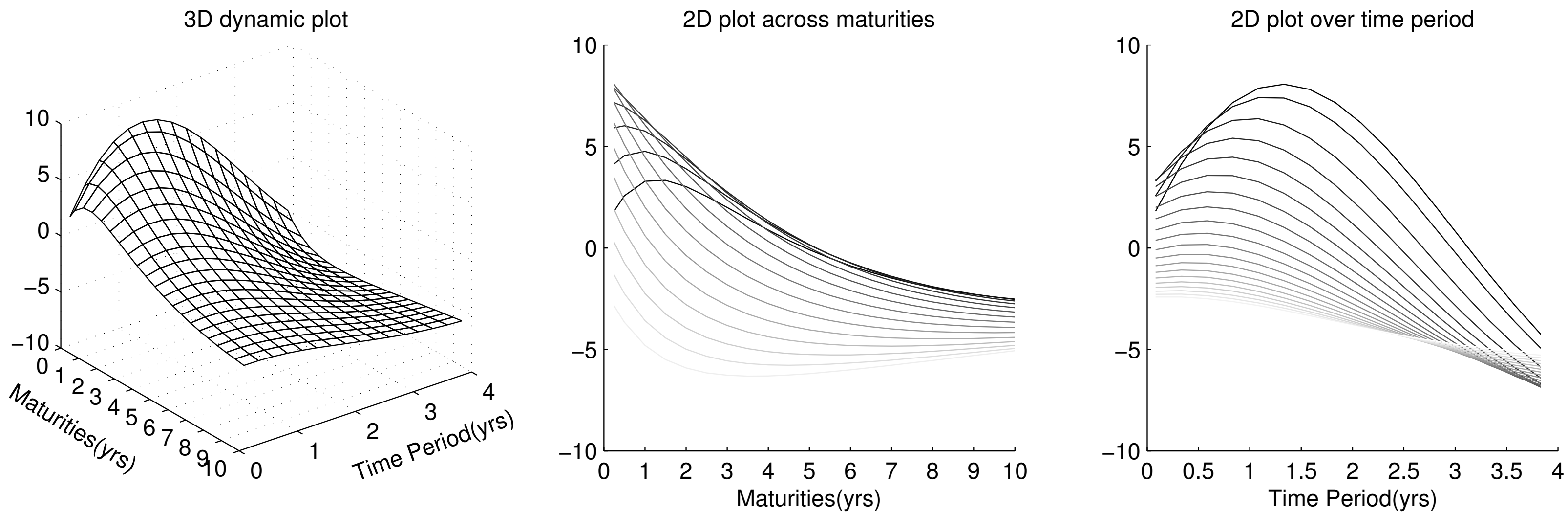
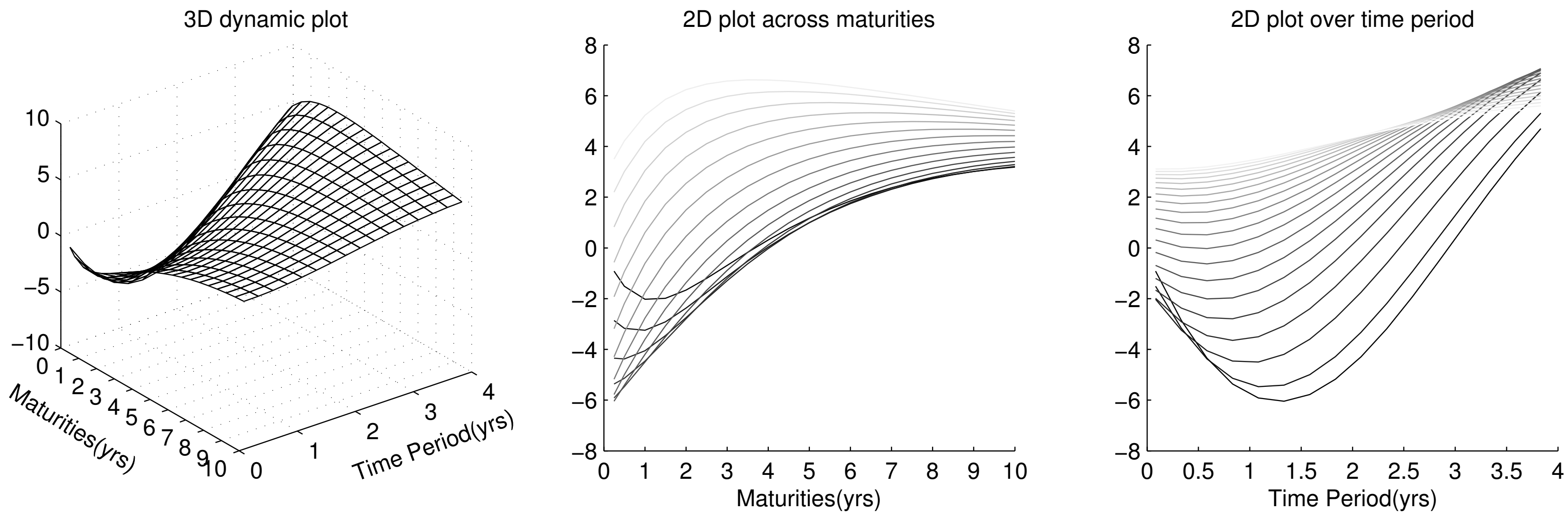


Figure 6.6: CPI of the QE3 announcement on the zero yield curve

This figure shows how the zero yield curve responds to the QE3 announcement on September 12 and 13, 2012 over a 4 yrs of time. (the left panel is 3D dynamic plot showing how the impact on the yield curve up to 10-yr maturity envolvs with n changes from 0 to 4; the middel panel is a 2D version of the left panel in which the darker(lighter) curve represents the impact on the whole yield curve when n is smaller(larger); the right panel is another 2D version of the left panel in which the darker(lighter) curve shows how the impact on the shorter(longer) maturity yield changes over the 4 yrs of time.



As we can see in the middle panel of Figure 6.3, the announcement of QE3 decreased the aesr_t^h for $h < 6$-yr, and increased aesr_t^h for $h > 6$-yr, this just mirrored the MEP's impact on aesr_t^h. This means that the market seemed to believe the QE3 would decrease the short term yields but increase the long term yields. From the right panel of Figure 6.3, we can also see that QE3 announcement actually increased the market uncertainty as the ytp_t^h increased for all horizons considered, this is also mirrored the MEP's impact on ytp_t^h. In the long run, the announcement was expected to push up the yield curve even further as shown in Figure 6.6: the whole yield curve was expected to increase by 4 bps or so in 4 years of time, although the short end was expected to decrease (by up to 6 bps) within the first 3 years.

Based on these three case studies: from the LSAP in 2008, the MEP in 2011, then to the QE3 in 2012, it seems that the announcements of the unconventional monetary policies has started to lose their effectiveness and to have unintended consequences. More solid assessment and justification of the unconventional monetary policies are definitely needed before more unconventionals are implemented.

7 Conclusions

In this paper, based on a three factor Gaussian dynamic term structure model, I develop a novel approach to construct legitimate measures of market expectation and term premia. As by-products of this approach, I also provide better measures for accessing state variables' generic impact on the yield curve. The novelty of my approach is calibrating the $\mathbb{P}$-measure parameters using a constraint optimization such that the resulting market expectations and term premia impact the forward curve correctly.

The main empirical observations include: a significant portion (40%) of a short rate shock has a uniform impact on the middle part and the long end of the yield curve, and the impact is also quite persistent; in the period of the conundrum during 2004-05 the drop in term premia not only happened at long maturities but also at short maturities despite of the increasing short maturity yield during that period; from late 2010 on the short maturity yield mainly consisted of the term premium, and it has been shrinking since mid of 2011; three case studies show that the unconventional monetary policies have started to lose their effectiveness and to have unintended consequences.

For future research, the next step would be to thoroughly inspect the statistical properties of the calibrated $\mathbb{P}$-measure parameters; another direction is to develop a more flexible market price of risk specification to capture the empirically observed unrestricted R^2 curves.

Appendices

A Derivations of CCI_t, UCI and CPI_t, UPI

A.1 When $k = m$

In this case the results are trivial:

$$\begin{aligned}
\text{CCI}_t(\delta, x, \Delta t, \mathbf{i}) &\equiv \mathbb{E}_t^{\mathbb{P}}\left(y_{t+\Delta t}(x) \middle| \Delta Z_{t+\Delta t} = \delta\right) - \mathbb{E}_t^{\mathbb{P}}\left(y_{t+\Delta t}(x)\right) \\
&= \frac{\int_0^x \mathbf{C}(s)\, ds}{x}\left[Z_t + \delta - \mathbb{E}_t^{\mathbb{P}}\left(Z_{t+\Delta t}\right)\right] \\
&= \frac{\int_0^x \mathbf{C}(s)\, ds}{x}\left[\delta + \left(I - \exp\left(\mathbf{A}^{\mathbb{P}}\Delta t\right)\right)(Z_t - \mu)\right]
\end{aligned}$$

$$\begin{aligned}
\text{UCI}(\delta, x, \Delta t, \mathbf{i}) &\equiv \mathbb{E}^{\mathbb{P}}\left(y_{t+\Delta t}(x) \middle| \Delta Z_{t+\Delta t} = \delta\right) - \mathbb{E}^{\mathbb{P}}\left(y_{t+\Delta t}(x)\right) \\
&= \frac{\int_0^x \mathbf{C}(s)\, ds}{x}\delta.
\end{aligned}$$

$$\begin{aligned}
\text{CPI}_t(\delta, x, \Delta t, n, \mathbf{i}) &\equiv \mathbb{E}_t^{\mathbb{P}}\left(y_{t+n}(x) \middle| \Delta Z_{t+\Delta t} = \delta\right) - \mathbb{E}_t^{\mathbb{P}}\left(y_{t+n}(x)\right) \\
&= \frac{\int_0^x \mathbf{C}(s)\, ds}{x}\exp\left(\mathbf{A}^{\mathbb{P}}(n - \Delta t)\right) \times \\
&\quad \left[\delta + \left(I - \exp\left(\mathbf{A}^{\mathbb{P}}\Delta t\right)\right)(Z_t - \mu)\right]
\end{aligned}$$

$$\begin{aligned}
\text{UPI}(\delta, x, \Delta t, n, \mathbf{i}) &\equiv \mathbb{E}^{\mathbb{P}}\left(y_{t+n}(x) \middle| \Delta Z_{t+\Delta t} = \delta\right) - \mathbb{E}^{\mathbb{P}}\left(y_{t+n}(x)\right) \\
&= \frac{\int_0^x \mathbf{C}(s)\, ds}{x}\exp\left(\mathbf{A}^{\mathbb{P}}(n - \Delta t)\right)\delta.
\end{aligned}$$

A.2 When $1 \le k < m$

First let's define a transfer matrix $M^{\mathrm{p}}(\mathbf{i})$ such that states in $Z_{\mathbf{i}}$ are collected in a partition with indexes of $\{1,2,\cdots,k\}$ in $Z^{\mathrm{P}} = M^{\mathrm{p}}(\mathbf{i})\,Z$ without altering the their original order in $Z_{\mathbf{i}}$ and $Z_{\bar{\mathbf{i}}}$, $\bar{\mathbf{i}}$ is the complement of $\mathbf{i}$ in $\{1,2,\cdots,n\}$. Z^{P} is partitioned as:

$$Z^{\mathrm{P}} = \begin{bmatrix} Z^{\mathrm{P}}_{\mathbf{1}} \\ Z^{\mathrm{P}}_{\mathbf{2}} \end{bmatrix},$$

where $\mathbf{1} \equiv \{1,2,\cdots,k\}$ and $\mathbf{2} \equiv \{k+1,k+2,\cdots,n\}$.

Conditional on information up to time t, $\Delta Z^{\mathrm{P}}_{t+\Delta t}$ has a multivariate normal distribution with

$$\begin{aligned}
\Lambda &\equiv \mathbb{E}^{\mathbb{P}}_t\left(\Delta Z^{\mathrm{P}}_{t+\Delta t}\right) = M^{\mathrm{p}}(\mathbf{i})\left(I - \exp\left(\mathbf{A}^{\mathbb{P}}\Delta t\right)\right)(\mu - Z_t) \\
\Omega &\equiv \mathbb{V}ar\left(\Delta Z^{\mathrm{P}}_{t+\Delta t}\right) \\
&= M^{\mathrm{p}}(\mathbf{i})\exp\left(\mathbf{A}^{\mathbb{P}}\Delta t\right)\left[\int_0^{\Delta t}\exp\left(-\mathbf{A}^{\mathbb{P}}s\right)\mathbf{B}\mathbf{B}^{\mathsf{T}}\exp\left(-\mathbf{A}^{\mathbb{P}}s\right)^{\mathsf{T}}ds\right]\exp\left(\mathbf{A}^{\mathbb{P}}\Delta t\right)^{\mathsf{T}}\left(M^{\mathrm{p}}(\mathbf{i})\right)^{\mathsf{T}}.
\end{aligned}$$

By definition we have

$$\begin{aligned}
\mathrm{CCI}_t(\delta,x,\Delta t,\mathbf{i}) &\equiv \mathbb{E}^{\mathbb{P}}_t\left(y_{t+\Delta t}(x)\,|\,\Delta Z_{\mathbf{i},t+\Delta t} = \delta\right) - \mathbb{E}^{\mathbb{P}}_t\left(y_{t+\Delta t}(x)\right) \\
&= \frac{\int_0^x \mathbf{C}(s)\,ds}{x}\left[\mathbb{E}^{\mathbb{P}}_t\left(Z_{t+\Delta t}\,|\,\Delta Z_{\mathbf{i},t+\Delta t} = \delta\right) - \mathbb{E}^{\mathbb{P}}_t\left(Z_{t+\Delta t}\right)\right] \\
&= \frac{\int_0^x \mathbf{C}(s)\,ds}{x}\times \\
&\quad \left[M^{\mathrm{p}}(\mathbf{i})^{-1}\,\mathbb{E}^{\mathbb{P}}_t\left(\Delta Z^{\mathrm{P}}_{t+\Delta t}\,|\,\Delta Z^{\mathrm{P}}_{\mathbf{1},t+\Delta t} = \delta\right) + \left(I - \exp\left(\mathbf{A}^{\mathbb{P}}\Delta t\right)\right)(Z_t - \mu)\right] \\
&= \frac{\int_0^x \mathbf{C}(s)\,ds}{x}\times \\
&\quad \left[M^{\mathrm{p}}(\mathbf{i})^{-1}\begin{bmatrix}\delta \\ \Lambda_{\mathbf{2}} + \Omega_{\mathbf{2},\mathbf{1}}\Omega^{-1}_{\mathbf{1},\mathbf{1}}(\delta - \Lambda_{\mathbf{1}})\end{bmatrix} + \left(I - \exp\left(\mathbf{A}^{\mathbb{P}}\Delta t\right)\right)(Z_t - \mu)\right],
\end{aligned}$$

$$
\begin{aligned}
\mathrm{CPI}_t\left(\delta,x,\Delta t,n,\mathbf{i}\right) &\equiv \mathbb{E}_t^{\mathbb{P}}\left(y_{t+n}\left(x\right)|\,\Delta Z_{\mathbf{i},t+\Delta t}=\delta\right)-\mathbb{E}_t^{\mathbb{P}}\left(y_{t+n}\left(x\right)\right)\\
&= \frac{\int_0^x \mathbf{C}\left(s\right)ds}{x}\exp\left(\mathbf{A}^{\mathbb{P}}\left(n-\Delta t\right)\right)\times\\
&\quad \left[M^{\mathrm{p}}\left(\mathbf{i}\right)^{-1}\begin{bmatrix}\delta\\ \Lambda_{\mathbf{2}}+\Omega_{\mathbf{2},\mathbf{1}}\Omega_{\mathbf{1},\mathbf{1}}^{-1}\left(\delta-\Lambda_{\mathbf{1}}\right)\end{bmatrix}+\left(I-\exp\left(\mathbf{A}^{\mathbb{P}}\Delta t\right)\right)\left(Z_t-\mu\right)\right].
\end{aligned}
$$

where the last equation is by the conditional distribution of multivariate normal random variables.

Again by definition, $\mathrm{UCI}\left(\delta,x,\Delta t,\mathbf{i}\right)$ and $\mathrm{UPI}\left(\delta,x,\Delta t,n,\mathbf{i}\right)$ are given by

$$
\begin{aligned}
\mathrm{UCI}\left(\delta,x,\Delta t,\mathbf{i}\right) &\equiv \mathbb{E}^{\mathbb{P}}\left(y_{t+\Delta t}\left(x\right)|\,\Delta Z_{\mathbf{i},t+\Delta t}=\delta\right)-\mathbb{E}^{\mathbb{P}}\left(y_{t+\Delta t}\left(x\right)\right)\\
&= \frac{\int_0^x \mathbf{C}\left(s\right)ds}{x}M^{\mathrm{p}}\left(\mathbf{i}\right)^{-1}\begin{bmatrix}\delta\\ \Omega_{\mathbf{2},\mathbf{1}}\Omega_{\mathbf{1},\mathbf{1}}^{-1}\delta\end{bmatrix},
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{UPI}\left(\delta,x,\Delta t,n,\mathbf{i}\right) &\equiv \mathbb{E}^{\mathbb{P}}\left(y_{t+n}\left(x\right)|\,\Delta Z_{\mathbf{i},t+\Delta t}=\delta\right)-\mathbb{E}^{\mathbb{P}}\left(y_{t+n}\left(x\right)\right)\\
&= \frac{\int_0^x \mathbf{C}\left(s\right)ds}{x}\exp\left(\mathbf{A}^{\mathbb{P}}\left(n-\Delta t\right)\right)M^{\mathrm{p}}\left(\mathbf{i}\right)^{-1}\begin{bmatrix}\delta\\ \Omega_{\mathbf{2},\mathbf{1}}\Omega_{\mathbf{1},\mathbf{1}}^{-1}\delta\end{bmatrix}.
\end{aligned}
$$

References

Adrian, T., R. K. Crump, and E. Moench (2013). Pricing the term structure with linear regressions. *Journal of Financial Economics*. 2

Ang, A. and M. Piazzesi (2003). A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary economics 50*(4), 745–787. 10

Bauer, M. D., G. D. Rudebusch, and J. C. Wu (2012). Correcting estimation bias in dynamic term structure models. *Journal of Business & Economic Statistics 30*(3), 454–467. 4, 9

Bernanke, B. S. (2006). Reflections on the yield curve and monetary policy. *Board of Governors of the Federal Reserve System (US) Speech* (Mar 20). 29

Brace, A., D. Gatarek, and M. Musiela (1997). THE MARKET MODEL OF INTEREST RATE DYNAMICS1. *Mathematical finance 7*(2), 127–147. 6

Brooke, M., N. Cooper, and C. Scholtes (2000). Inferring market interest rate expectations from money market rates. *Bank of England Quarterly Bulletin, November 2000*. 2

Cochrane, J. H. and M. Piazzesi (2005). Bond risk premia. *The American Economic Review 95*(1), 138–160. 3

Cochrane, J. H. and M. Piazzesi (2008). Decomposing the yield curve. *Graduate School of Business, University of Chicago, Working Paper*. 3, 10, 29

Cook, T. and T. Hahn (1990). Interest rate expectations and the slope of the money market yield curve. *FRB Richmond Economic Review 76*(5), 3–26. 3

Dai, Q. and K. Singleton (2000). Specification analysis of affine term structure models. *Journal of Finance 55*(5), 1943–1978. 6, 7

Duffee, G. (2002). Term premia and interest rate forecasts in affine models. *Journal of Finance 57*, 405–443. 7

Duffee, G. R. and R. H. Stanton (2012). Estimation of dynamic term structure models. *The Quarterly Journal of Finance 2*(02). 4, 8

Heidari, M. and L. Wu (2009). A joint framework for consistently pricing interest rates and interest rate derivatives. *Journal of Financial and Quantitative Analysis 44*(03), 517–550. 8

Joslin, S., M. Priebsch, and K. Singleton (2010). Risk premiums in dynamic term structure models with unspanned macro risks. *Manuscript, Stanford University, September*. 29

Joyce, M., J. Relleen, and S. Sorensen (2008). Measuring monetary policy expectations from financial market instruments. *Bank of England working papers*. 2

Kim, D. and J. Wright (2005). An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates. *FEDS Working Paper*. 2, 29

Kim, D. H. and A. Orphanides (2012). Term structure estimation with survey data on interest rate forecasts. *Journal of Financial and Quantitative Analysis 47*(01), 241–272. 3

Koop, G., M. Pesaran, and S. Potter (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics 74*(1), 119–147. 13

Li, H. and X. Ye (2012). A unified heath-jarrow-morton approach to gaussian dynamic term structure models. Working paper, Risk Management Institute, National University of Singapore. 5

Lu, B. and L. Wu (2009). Macroeconomic releases and the interest rate term structure. *Journal of Monetary Economics 56*(6), 872–884. 10

Pesaran, H. H. and Y. Shin (1998). Generalized impulse response analysis in linear multivariate models. *Economics letters 58*(1), 17–29. 13

Piazzesi, M. (2005). Bond yields and the federal reserve. *Journal of Political Economy 113*(2), 311–344. 26

Piazzesi, M., J. Salomao, and M. Schneider (2013). Trend and cycle in bond premia. Working paper, Stanford University, Department of Economics. 3

Söderlind, P. and L. Svensson (1997). New techniques to extract market expectations from financial instruments. *Journal of Monetary Economics 40*(2), 383–429. 2

Wu, L. and F. X. Zhang (2008). A no-arbitrage analysis of macroeconomic determinants of the credit spread term structure. *Management Science 54*(6), 1160–1175. 10