# Optimal Corporate Hedging Using Options with Basis and Production Risk


**Abstract**

We investigate the optimal hedging strategy for a firm using options, where the role of production and basis risk are considered. Contrary to the existing literature, we find that the exercise price which minimizes the shortfall of the hedged portfolio is primarily affected by the amount of cash spent on the hedging. Also, we decompose the effect of production and basis risk showing that the former affects hedging effectiveness while the latter drives the choice of the optimal contract. Fitting the model parameters to match a financial turmoil scenario confirms that suboptimal option moneyness leads to a non-negligible economic loss.



JEL Classification: G30; G32.

Keywords: Risk management; Option hedging; Expected shortfall.

# 1. Introduction

The two major global crises—the default of subprime mortgages and the European sovereign debt downfall—that have characterized recent years have produced, among other effects, a dramatic soaring of volatility. In these circumstances, it is fundamental for firms to have effective hedging strategies set up so as to avoid the disruptive consequences of price jumps. As evidence of its relevance, over the last 30 years risk management has attracted the attention of a large body of financial literature, which has investigated both the theory and the practice of firm hedging. Among these studies, scholars have investigated theoretical hedging motivations, the empirical determinants for firms to protect against market risks and the type of instruments—mainly, financial derivatives—used to this purpose. Within the last topic, firms can basically opt between linear (e.g., forwards, futures and swaps) and non-linear (e.g., options) instruments. The optimality of the one type relative to the other has been discussed thoroughly in the literature, as well as the optimal hedging strategy when linear instruments are chosen. Surprisingly, in spite of its widespread use in firm hedging strategies, little research has been conducted on the determination of an optimal hedging policy using options.[1]

In this paper we aim at shedding light on how firms can optimally choose an option contract so as to minimize the inherent risk exposure. As risk management with options—unlike linear financial derivatives—involves a hedging cost, the investigation of this optimality requires a constraint given by the firm's budget. Also, the "optimality" itself needs to be somehow defined. In the case of linear hedging, as for futures, the most common approach is to minimize the variance of the hedged position, computing the so-called minimum variance hedge ratio (Johnson, 1960; Stein, 1961; Ederington, 1979). However, this approach is not applicable to option hedging, as the hedged position—that is the combined position of the exposure (i.e., the naked position) and the derivatives—is very different. In the event of linear hedging, the hedged position does not have a directional risk, as the potential gains (losses) from the derivatives offset the losses (gains) on the naked position, leaving

[1] Bodnar *et al*. (1998) document that among derivative users, 68 percent of US firms use options.

the hedger with virtually no uncertainty.[2] Through option hedging, however, firms may set a floor to the losses (i.e., neutralizing the downside risk), but the right to exercise the derivative contract leaves the potential upside unaffected. Hence, the hedged position is asymmetric and the hedging efficiency cannot be evaluated through the conventional variance.

Based on this reasoning, Ahn *et al*. (1999) propose a value-at-risk (VaR) approach to option hedging. The authors consider a firm facing only price risk (i.e., the uncertainty of future price), since the production is taken as certain and the spot-underlying asset correlation is unitary.[3] In this framework, the optimal option hedging strategy is represented by the contract (i.e., the put option strike price) that minimizes the hedged position VaR. However, it has been largely shown (Moschini and Lapan, 1995; Brown and Toft, 2002) that the presence of a non-hedgeable (production) risk creates a convex exposure that is more efficiently hedged *via* non-linear instruments. In other words, the presence of quantity (production) risk is the main theoretical justification for the use of options in a hedging strategy. Analogously, basis risk, i.e. the non-unitary correlation between the spot position and the asset used for hedging, can hardly be ignored in a risk management strategy, as it is embedded in virtually any hedging instrument. Finally, the choice of VaR as a measure of risk in the hedging framework is arguable. Since VaR is a single percentile of the future position probability distribution, it provides no information on the size of the loss exceeding such a level and therefore is unable to capture tail risk.[4] In corporate risk management the worst case scenario may be "bad" enough to liquidate the company, and VaR simply would be unaffected. Also, as it has been largely documented (Artzner *et al*., 1999), VaR is not subadditive, meaning that the VaR of aggregated positions may be larger than the sum of the individual VaRs. This is counterintuitive: different corporate risks may enjoy natural diversification due to anticorrelation, and the risk of the global position is expected to be smaller than the sum of the risks.

[2] However, such a perfect hedge is a pure theoretical concept, as in practice the hedger with futures always trades off price risk with basis risk.

[3] The authors assume that the firm enters in a derivative contract written on the exact same type of asset exposed to risk (or an asset having perfect correlation).

[4] Provocatively, Szegö (2002) affirms that "*VaR does not measure risk*," as using VaR for risk measurement purposes is like "*measuring the distance between two points using a rubber band instead of a ruler*."

In this paper we determine the optimal hedging strategy for a firm using option contracts, explicitly considering the role of production (quantity) and basis (proxy) risk. Furthermore, we employ the expected shortfall (ES)—known also as conditional VaR—as a risk measure. The choice of this different metric carries a number of advantages relative to VaR. First, the ES considers both the size and the probability of experiencing losses greater than a given level. Second, the ES enjoys subadditivity (Rockafellar and Uryasev, 2000; Acerbi and Tasche, 2002).[5] These motivations explain why the most recent risk management literature, regulators and the financial industry are rapidly abandoning the simplistic VaR methodology in favor of more accurate measure such as the ES. For instance, very recently the Basel Committee on Banking Supervision suggested switching the bank quantitative risk metrics system from VaR to ES.[6]

The main contribution of this study is to be the first—to the best of our knowledge—to analyze the role of production (quantity) risk, along with basis (proxy) risk, in a framework of corporate option hedging. While the importance of using options for risk management when quantity is uncertain has been widely discussed, no prior investigations have been carried out to understand how options should be used to maximize the hedging effectiveness. Likewise, although any hedging strategy employing traded derivatives inevitably leads to a partial mismatch between the naked and the hedging positions (i.e., basis risk), in contrast to literature on futures hedging, the impact of basis risk on the optimal option contract has not been documented yet. As a second contribution, this study employs the expected shortfall as a tail risk measure in a context of corporate hedging. The increasing relevance of this metric and the dominance relative to VaR—for both practical and regulatory purposes—requires a study of the optimal hedging policy when we abandon the unrealistic assumption that risk is measurable by a simple (negative) outcome, as VaR suggests, instead of considering the size of the adverse tail of the outcome distribution (expected shortfall).

5 An empirical application showing that ES is coherent and hence a risk measure superior to VaR for hedge funds' performance is Liang and Park (2007).

6 On p. 3 of the *Fundamental review of the trading book*, dated May 2012, one can read: *"A number of weaknesses have been identified with using value-at-risk (VaR) for determining regulatory capital requirements, including its inability to capture 'tail risk.' For this reason, the Committee has considered alternative risk metrics, in particular expected shortfall (ES)."*

In terms of results, our model offers interesting insights, originating from the inclusion of the two additional sources of uncertainty along with the use of expected shortfall. First, we find evidence that the optimal hedge is greatly influenced by the amount of cash spent for the hedging program, even in a framework with no quantity or proxy risk. This conclusion is striking, as Ahn *et al*. (1999) show that, in a VaR framework, the optimal option contract is independent of the hedging budget. Second, our model documents the separate effect of basis and production risk. When only the latter is in place, we observe a dramatic reduction of hedging effectiveness, proving its major role in option hedging. The former, instead, notably affects the optimal contract (i.e., the put option exercise price), as a change of one basis point in the correlation between the spot position and the option underlying asset yields approximately a 0.27 percent variation on hedging effectiveness. In terms of optimal contract, firms respond to an intensification of basis risk by increasing the moneyness of the hedge, leaving a smaller fraction of its price risk unexposed. By contrast, if spot-quantity correlation highly affects the functional shape of the risk exposure, hence justifying the option use for hedging purposes, it is only important to a limited extent for detecting the optimal price protection level.

The remainder of the paper is organized as follows. The next section provides a theoretical motivation of option hedging, as proposed in the financial literature. Section 3 develops a framework in which the firm faces both price and quantity risk and hedges the naked position through option contracts. Section 4 proposes a simulation based on price and production data for a widespread agricultural commodity and discusses the main results. Finally, section 5 concludes.

## 2. Risk management with options

The non-linearity of the corporate risk exposure motivates the use of a non-linear hedge. In fact, if the firm's position at risk is not a linear function of price, entering into linear instruments determines over-hedging, i.e. payoffs resulting from the purchased (sold) derivatives are higher than the gain or loss on the naked position. Options, due to the non-linearity of their payoff, can instead match the shape of the exposure function.

Since the seminal contribution of Adler and Detemple (1988), the literature has offered three main reasons to explain the convex (concave) shape of the risk exposure.

First, the non-linear exposures may be a result of a (non-zero) correlation between non-hedgeable (short-selling and borrowing constraints) and hedgeable (price) risks. In Adler and Detemple's (1988) model, where risk-averse managers are subject to limited access to financing and to short-selling constraints, firms suffering from borrowing constraints and high price risk are predicted to be more likely recurring to option hedging.

A second possible explanation is proposed by Froot *et al*. (1993), as non-linear derivatives allow firms to more accurately coordinate investment and financing strategies. If external finance is more costly than internally generated funds, firms have the incentive to undertake hedging strategies so to reduce the states of nature in which they will need to raise new capital. The choice of recurring to hedging programs will also depend on the correlation between the internal cash flows and the investment opportunities relative to market prices. When the sensitivities are different, the firm exposure to market risk becomes non-linear, making the use of options more efficient. Adam (2002) extends Froot *et al*.'s model (1993) to an intertemporal framework and focuses on the cost differential between internal and external capital. His model predicts that less (more) financially constrained firms will buy (sell) options in order to hedge the convex (concave) risk exposure. In an empirical study of the gold mining industry, Adam (2009) finds support for the implications of Froot *et al*. (1993) and Adams (2002). In particular, firms with high capital expenditures are more active option users, as well as large and less financially constrained companies.

The third and probably most cited reason for the non-linearity of the risk exposure is the correlation between the production and the price risk. While Lapan *et al*. (1991) show that optimal hedge for a firm facing only price risk is ensured by forward (linear) contracts, as options (non-linear) become redundant, Sakong *et al*. (1993) demonstrate that the opposite is true when the firm has to also cope with production uncertainty. Moschini and Lapan (1995) reach similar conclusions, showing that optimal hedging policy includes options if hedgeable and non-hedgeable risks are correlated. Brown and Toft (2002) suggest the same arguments by deriving the hedging strategy which maximizes the firm's value, subject to financial distress costs. The authors show that the more effective strategy depends on the correlation between price and quantity, their volatilities and the profit margin. If the correlation is negligible, forward (linear) strategies outperform option (non-linear) approaches. A negative correlation motivates instead for the use of plain-vanilla options, whilst with a positive correlation firms are better off with exotic derivatives. Furthermore, the higher the quantity risk and the price-quantity correlation, the more using options is justified. By contrast, Adam (2009), based on his sample of 69 gold mining companies, finds little empirical support, as he concludes that the role of production uncertainty moderately explains the use of option hedging.[7] However, as production risk in the gold industry is modest, this empirical setting is not the perfect test to validate this conclusion.[8] Accordingly, with limited or no production risk, linear instruments are more suitable than options, and this in turn justifies the widespread use of forwards, futures and swaps for hedging purposes.

Among the other rationales for option hedging, Bartram (2006) suggests that the accounting effects of the gains and losses of derivatives may play a role. When firms use linear instruments to hedge their market risks, the payoffs of the derivative position are intended to offset those originated by the exposure. In the event of a favorable market price movement, firms experience an increase of revenues (or a decrease of costs), which is counterbalanced by a loss on the hedge position. CFOs may therefore face difficulties justifying to shareholders the losses on the position of their derivatives and the missed opportunities to increase earnings. Instead, options only involve the payment of an upfront premium—which is amortizable over time—that insures the firm against downside risk, preserving the potential upside from a positive price movement.

With regards to the derivation of an optimal option hedging policy, Ahn *et al*. (1999) propose an approach based on the minimization of the VaR of the hedged

[7] The full sample consists of 111 firms, but the authors are only able to provide data on production variability for 69 of these.

[8] The standard deviation of the world gold production from 1990 to 2009 is only 3.22 percent (production data is obtained from 2010 US geological survey).

position. Option hedging necessarily leads to a trade-off between the effectiveness of the insurance (the amount of hedged risk) and its cost (the option premium). Interestingly, the authors show that, for reasonable levels of the expense which the firm devotes to the hedging program, the optimal option contract (i.e., the put option strike price) is independent of the hedging cost. This finding, which is an important insight, holds under the assumption of absence of production risk and when VaR is recognized to be the proper risk measure.

# 3. Optimal hedging contract

A non-linearity of the naked position represents an issue for many types of firms exposed to market risks. It is not by chance that some of the seminal contributions originate from the agricultural risk-management literature (Moschini and Lapan, 1992, 1995; Lapan *et al.*, 1991), as production uncertainty is a major source of risk. For instance, a farmer can easily hedge the price risk for a given quantity, but it is much more difficult to anticipate and hedge the produced quantity. Non-linear exposures also arise when revenues or costs depend on the change of exchange rates. Stulz (2003) suggests that if cash flows are expressed in a foreign currency, the depreciation of the home currency produces a contemporary increase of domestic-denominated cash flow and a higher number of units sold abroad (due to the increased competitiveness). This double effect makes the exposure a non-linear function to the underlying price (the exchange rate). Similarly, exposures are no longer linear if exporting firms adopt pricing-to-market strategies—to gain market share—during the phases of foreign currency appreciation.[9] If a non-linearity of the risk exposure motivates the use of options for hedging purposes, a clear understanding of how these derivatives should be used to minimize the risk of the hedged position is still an unaddressed issue and the scope of the next section.

## 3.1 The hedging model

In order to derive the optimal hedging strategy we set up a model of risk management, where the firm faces both proxy and quantity risk. We assume that the

[9] Conversely, Knetter (1994) argues that quantitative trade restrictions or capacity limitations can push exporting firms to make use of more aggressive pricing-to-market practices during foreign currency depreciation.

firm currently (at time $t$) expects to produce a quantity $Q_t$ of a cash asset whose known price is $S_t$. Accordingly, the current naked position of the firm—assumed long, without loss of generality—is given by the product of $S_t$ and $Q_t$. The future value of the naked position at the hedging horizon (time $T$) is instead uncertain. The gain or loss on the naked position at the hedging horizon $T$ is therefore the difference

$$Q_T S_T - Q_t S_t \exp[r(T-t)], \tag{1}$$

where we denote with $r$ the risk-free interest rate for the period $(T-t)$.

Similarly to Ahn *et al*. (1999), we impose the firm being subject to a budget constraint due to the maximum expenditure on the hedging program $(C)$[10]. Based on this constraint, to hedge the potential downside risk the firm buys a fraction $h < 1$ of one put option per unit of underlying asset, resulting in a total of $hQ_t$ put options expiring at time $T$.[11] Unlike previous studies (Ahn *et al*., 1999; Deelstra *et al*., 2010), we do not impose that the put option be written on the same underlying asset of the naked position. In practice, this is in fact an unrealistic assumption, since it is unlikely that the correlation between the naked position and the hedge position is perfect. For option hedging, this non-unitary correlation stems also from the fact that many risk sources (e.g., in commodity investing) can be hedged only through futures options, as options written on cash assets are not traded. The weaker this correlation, the more basis risk is residual in the hedge. Denoting with $S'_t$ the asset underlying the put option, we may write the gain or loss at maturity on the hedge position as

$$hQ_t \max(k - S'_T, 0) - C \exp[r(T-t)], \tag{2}$$

[10] It can be argued that hedging budgets are infrequently fixed in Dollar terms. However, replacing the maximum expenditure on the hedging program, $C$, with a positive function of the naked position (e.g., $C = cQ_tS_t$, with $c$ between 0 and 1) does not alter the setup and its conclusions.

[11] Using a single option rather than a weighted combination of contracts yielding the same exercise price is always optimal due to the convexity of the payoff function. Moreover, the constraint $h < 1$ is not binding for reasonable hedging expenses (Ahn *et al*., 1999).

where $k$ denotes the put option exercise price and $C$ the budget constraint, with $C = hQ_tP(t, S'_t)$ ($P$ denotes the current price, i.e. at time $t$, of a put option written on $S'_t$).[12,13]

The hedging problem consists of determining the optimal hedge ratio, that is $h$ that minimizes the risk of the gain or loss on the hedged position expressed as the expected shortfall (at a given confidence level, $\alpha$) of[14]

$$H_T = Q_TS_T - Q_tS_t\exp[r(T-t)] + hQ_t\max(k - S'_T, 0) - hQ_tP(t, S'_t)\exp[r(T-t)]. \quad (3)$$

The probability distribution of the gain or loss of the hedged position at time $T$ is necessary in order to compute the expected shortfall. Therefore, the stochastic dynamics of $S_t$, $S'_t$ and $Q_t$ have to be assumed, along with their correlation matrix. For simplicity, we use log-normal dynamics for both the assets and the quantity, i.e.:

$$S_T = S_t\exp\left[(\mu - .5\,\sigma^2)(T-t) + \sigma\varepsilon\sqrt{T-t}\,\right],$$
$$S'_T = S'_t\exp\left[(\mu' - .5\,(\sigma')^2)(T-t) + \sigma'\varepsilon'\sqrt{T-t}\,\right],$$
$$Q_T = Q_t\exp\left[\left(\mu_Q - .5\,\xi^2\right)(T-t) + \xi\varepsilon_Q\sqrt{T-t}\,\right],$$

and $\varepsilon, \varepsilon'$ and $\varepsilon_Q$ are standard correlated normal random variables with correlation matrix equal to

$$\mathrm{corr}\begin{bmatrix}\varepsilon\\ \varepsilon'\\ \varepsilon_Q\end{bmatrix} = \begin{bmatrix}1 & \rho & \theta_1\\ \rho & 1 & \theta_2\\ \theta_1 & \theta_2 & 1\end{bmatrix}.$$

Given the log-normal dynamics of $S'_t$, the current price of the put option written on $S'_t$ is provided by the usual Black-Scholes-Merton equation. Finally, the cumulative distribution function of the hedged position at the future time $T$ is used in order to compute the ES at the confidence level $\alpha$, integrating the quantiles of the hedged

[12] This constraint should be better read as $C \geq hQ_tP\left(t, S'_t\right)$, but *de facto* we can replace the inequality with an equality, as it is always rational to fully spend the budget in order to obtain a better hedge (Ahn et al., 1999).

[13] In the case of commodity hedging, as options are generally written on futures, $S'_t$ represents the futures price.

[14] The budget constraint $C = hQ_tP(t, S'_t)$, along with the limitation $h < 1$, delivers a one-to-one relationship between the optimal hedge ratio, $h$, and the optimal put exercise price, $k$. Therefore, the hedging problem can be identically presented as determining the put option exercise price which minimizes the expected shortfall of the hedged position.

portfolio on the probability distribution left tail. Formally, the optimization problem can be written as

$$\min_{h,k} \mathrm{ES}_\alpha (H_T) \text{ s.t.}$$
$$h < 1 \text{ and } C = hQ_t P(t, S'_t), \qquad (4)$$

where $\mathrm{ES}_\alpha = -\frac{1}{\alpha}\int_{-\infty}^{\alpha} q_s(H_T)dF_s(H_T)$ is the expected shortfall of $H_T$, that is the conditional expectation limited to the $s$-quantiles of $H_T$, $q_s(H_T)$, with $s \leq \alpha$.

Three observations are in order here. First, assuming log-normal variables is a simplifying assumption as it allows using Black and Scholes' (1973) option pricing model. Although for most financial assets we empirically witness a departure from this assumption, our choice does not meaningfully alter the insights we will later discuss. Second, these price dynamics are fairly general, as they can embrace the case of futures options, very common in the practice of hedging. In this instance, $S'_t$ denotes the current price of futures contract expiring past $T$, $\mu' = \mu - r$ and Black and Scholes (1973) equation is replaced by Black (1976) formula. This setup is also suitable to commodity price modeling, provided that the convenience yield $y$ is deterministic—in this case, $\mu = g - y$, with $g$ the unobservable asset growth rate. A last and important comment is related to the methodology employed to solve the optimization problem given in equation (4). Since no analytical cumulative probability distribution for $H_T$ is derivable, we recur to a Monte Carlo simulation, without any loss in the reliability of our findings, as shown in Bajo *et al.* (2013).[15]

### 3.2 An illustrative example

In order to illustrate the derivation of the optimal hedging contract presented in the previous paragraph, we consider the case of a dairy farm concerned about the future value fluctuation of its production.[16] We assume that, according to the cash budget, the farm expects to produce $Q_t = 5$ million hundredweights (CWT) of milk over the

[15] The authors present a semi-analytical approach for this optimization problem, representing the cumulative probability distribution of $H_T$ *via* a bivariate copula function with the given dependence structure.

[16] As noted, given the characteristic non-linear exposure produced by the correlation quantity-price when cash flows are expressed in a foreign currency, this example can be easily extended to the importer or exporter case.

next 6 months ($T - t = 0.50$), while the current price is $S_t = \$20/\text{CWT}$. Based on the farm's estimates, the naked position is therefore equal to $Q_t S_t = \$100$ million.

With no hedging taking place (i.e., setting $h = 0$), the farm is subject to a downside risk—expressed in terms of the average loss exceeding the first percentile of the naked position probability distribution—equal to \$26.7 million.[17] Instead, we assume that the farm is willing to spend a maximum amount equal to $C = \$1$ million—i.e. 1 percent of the cash position—to hedge its position against the possible decline in both the price and production of milk. The resulting gain or loss, combining both the naked and the hedge position at time $T$, is therefore (in \$ million, and choosing $r = 1.0$ percent)

$$Q_T S_T - 101.76 + 5h \max(k - S'_T, 0) - 5hP(t, S'_t) \times 1.01.$$

If the firm chooses an at-the-money (ATM) hedge, given the option cost (\$0.9708), the maximum amount of purchasable insurance is 1.03 million CWT, which represents a 20.1 percent fraction of the risk exposure ($h = 0.2060$). From equation (3), we know the ES is now roughly less than \$21 million (20.8), meaning that the firm has reduced its risk from the \$26.7 million of the unhedged position to a relatively safer level (around a \$6 million drop in the ES). However, while ATM options allow a good level of price insurance, as any drop below the current milk price is protected by the put option, they also generate some degree of quantity under-hedging due to the budget constraint. In fact, in the ATM hedge, the firm has left 79.9 percent of its expected production unprotected. In order to solve the trade-off between price-quantity protection which has arisen from the presence of a budget constraint, we solve the optimization problem described in equation (4). Figure 1a, depicting the ES (in \$ million) relative to the option moneyness, clearly shows the existence of this trade-off.

**Please insert Figure 1 about here**

[17] This result holds with the following parameterization: $\mu = 2.88$ percent, $\sigma = 20.39$ percent, $\mu_Q = 14.60$ percent, $\xi = 1.50$ percent, $\theta_1 = -26.61$ percent. The following section details the data source and how these values have been estimated.

The optimal strike is roughly 0.87, meaning that the minimization of the firm's ES is reached, leaving 13 percent of the price risk unhedged.[18] The Figure also indicates that the risk reduction is extremely significant. The optimal out-of-the-money (OTM) option contract produces a \$12 million ES, roughly \$9 million lower than the ATM option, showing that the choice of a proper strike price in option hedging assumes a great relevance. The same conclusion can be drawn analyzing Figure 1b, where the firm's ES is simply scaled by the same metric computed on the naked position, thus providing a relative and non-monetary measure of the borne risk after hedging. Referring to the previous numbers, an ATM option allows the farm to slightly reduce its ES, from \$26.7 to \$20.8 million. The post-hedging ES is therefore 77 percent of the unhedged position, suggesting a 23 percent reduction. The 0.87 OTM optimal contract guarantees the firm a 45 percent ES—relative to the unhedged case (\$12/\$26.7)—suggesting a 55 percent risk reduction. In differential terms, the opportune choice of the strike allows a 22 percent ES decrease.[19]

## 4. Data and results

In this section we apply the proposed model to the problem of estimating the optimal hedge ratio for a hypothetical firm facing both basis and production risk and willing to hedge using option contracts. We take the case of a producing firm having a long risk exposure on a commodity. In order to obtain a realistic outcome, we estimate the model parameters using a widespread agricultural commodity.[20] To hedge a naked position the firm needs a put option written on an underlying asset correlated with the asset held cash. For the sake of simplicity, and since futures prices are liquid and widely traded, we choose the nearby futures as a proxy of the asset underlying the put option.[21] Since the correlation between spot and futures prices is all but unitary,

[18] We define as moneyness the ratio between the put option exercise price and the current price of the underlying asset. Thus, an out-of-the-money (OTM) put option approaches the at-the-money (ATM) level as the moneyness ratio moves toward 1.0.

[19] The ES as a percentage of the unhedged ES is an inverse measure of hedging effectiveness, as it indicates the percentage of non-eliminated ES after hedging. Clearly, its complement is the fraction of eliminated risk, which is the traditional metric for measuring hedging effectiveness.

[20] As previously noticed, the problem can be extended to a firm facing uncertainty on the future cash flows denominated in a foreign currency.

[21] For the purpose of applying our model, it is fundamental that the spot position is hedged using a different asset, in order to generate basis risk. The liquidity of futures contracts allows us to present a clean test, our results not being affected by any type of market imperfections.

even considering the very same commodity, we have a broad representation of basis risk in our sample. The imperfect correlation between spot and futures prices is well documented in Table 1, where some descriptive statistics of a sample of the most common food and agricultural commodities are presented.[22] In order to be included in our sample, we impose the contemporaneous fulfillment of the following conditions: (a) presence of a futures contract written on the same commodity and listed at CBOT; (b) continuous series of spot prices and production; (c) minimum of 15 years of futures prices data. We collect yearly prices and total production from the US Department of Agriculture (USDA), National Agricultural Statistics Service (NASS) and futures prices from Thomson Reuters Datastream. The sample period ranges from 1995 through 2010.

**Please insert Table 1 about here**

Table 1 reports the pairwise spot/futures/production correlations, along with standard descriptive statistics of the yearly returns.[23] Starting the discussion with the commodity we have used in our illustrative example, i.e. milk, we notice that annual spot and futures returns do not differ much both in mean and median terms. Over the considered time period, this commodity has not experienced a significant price trend. However, a producer should be concerned about the degree of variability rather than the simple price direction. Both spot and futures returns have exhibited roughly 20 percent annual volatility, which is comparable to some risky financial asset such as long-dated bonds or moderately risky stocks. However, even in the presence of relatively high volatility, a hedger can still reduce the uncertainty of the spot position if a derivative contract written on a highly correlated asset is available. For the case of milk, the extremely high spot-futures correlation (0.98) ensures a moderate basis risk and therefore a modest uncertainty of the hedged position. In terms of production risk, Table 1 shows very low variability of annual milk

[22] Each commodity is sorted by the total value of US production (year 2010).

[23] The low number of observations limits the statistical significance of reported coefficients, but the computation of these statistics has the only purpose of producing reasonable parameters for our hedging model. In any case, reported parameters are fairly stable when we estimate them over larger window periods (for those commodities showing data availability) and later in the paper we present a sensitivity analysis in order to investigate the effects of deviations from the assumed parameters.

production (1.5 percent), which places the producer in the very fortunate situation of virtually no quantity risk. Combining the small quantity risk with the very limited basis risk we may treat the case of milk as a benchmark, in which the imperfections of the hedge are nearly absent. A similar pattern of low quantity risk is evident for cattle and hogs. As for milk, the production is based on the stock farming and therefore the output is uncertain to a very limited extent. By contrast, other commodities exhibit more variability: corn, rice and soybeans have approximately 10 percent annual volatility, whilst cotton more than twice as much. In these circumstances, with a significant production risk in place, the correlation between quantity and prices also becomes relevant. In fact, the higher price-quantity (negative) correlation, the more the exposure takes on a convex shape.

In terms of spot-futures correlation, Table 1 shows large differences among the considered commodities. If the milk spot price is highly correlated with the corresponding futures (0.98), in other cases the low correlation raises doubts about the benefit and effectiveness of a potential hedging strategy. For instance, a rice spot-futures correlation is 0.75, but cotton and soybeans show lower values (0.47). Hence, for our sample of commodities, the role of basis and production risk plays a very different role. The choice of milk for the illustrative example is based upon the specific characteristics of this commodity, i.e. the virtual absence of both risks. In such a setting, the model derives the optimality of an option contract when the hedger faces only price risk, and in this sense the results are somehow comparable to Ahn *et al*. (1999), even if the different risk measure offers interesting insights. For instance, *Ahn et al*. (1999) state that the optimal strike price does not depend on the allocated hedging budget. By contrast, we find that when tail risk is considered, as in the case of the expected shortfall, the optimal option contract is greatly influenced by the amount of cash spent for the hedging program.[24]

The main contribution of our analysis relies upon the inclusion of basis and quantity risk. Hence, milk represents a benchmark, but the remainder of the discussion builds on a commodity characterized by an appreciable level of production uncertainty (proxy for a firm's quantity risk) and non-perfect spot-

[24] For the sake of brevity we do not show the ES sensitivity to the hedging budget for the milk case. However, this result is available upon request.

futures correlation (proxy for basis risk). Wheat is a suitable choice given its parameters: the correlation between spot and futures returns is 0.91, allowing some basis risk but at the same time hedging effectiveness, while the production volatility is about 15 percent, approximately ten times the quantity risk associated with milk.[25,26]

In presenting our findings, the remaining part of the paper will not refer to the ES in dollar terms, but to its scaled transformation (as in Figure 1b).[27] This normalization does not affect the contract optimality obtained *via* ES minimization, and it gives a relative and more informative measure of the risk borne by the firm. This measure is also an inverse proxy of hedging effectiveness, representing the share of ES not hedged away.

**Please insert Figure 2 about here**

Figure 2a displays the ES for the case of wheat. The first insight relates to the lower level—relative to the benchmark case of milk—of hedging effectiveness. At the optimal point, the percentage of tail risk reduction is 12 percent (*vs*. 55 percent in the milk example).[28] This notable difference in the hedging performance is expected, as it stems from the contemporaneous presence of basis and quantity risk, which produces a higher uncertainty over the firm's position.

More importantly, we note that the contract optimality is not very dissimilar from the milk case, as the optimal strike price is approximately 0.88 times the current spot price (indicating a 12 percent spot position left unhedged). Unlike the base case, the economic relevance of the optimal contract seems at first sight less

[25] The volatility of the firm's production is presumably higher than the standard deviation of the US total crop, as the overall production benefits from a "diversification" effect. Some areas of the US may suffer some production drop, being compensated by other states where the climate has been more favorable. However, the sensitivity of the production volatility over the contract optimality is investigated and it will be presented in the next section of the paper.

[26] We do not report the results on the hedging simulation for the other commodities we have earlier presented, as they do not add incremental insights to the discussion. Also, we later present a sensitivity analysis which *de facto* covers the same parameters of the unreported commodities.

[27] The expected shortfall is divided by the same metric computed on the unhedged position.

[28] The low level of hedging effectiveness might to some extent be surprising to the reader. However, the expected shortfall reduction, as a measure of tail risk, is not comparable to the correspondent metric used for computing hedging effectiveness in futures hedging (variance of the hedged position). The amount of hedging effectiveness presented in this paper is consistent with the figures shown in Ahn *et al*. (1999).

important, as the benefit of selecting the proper strike price relative to an ATM-hedge leads to a 4 percent increase in the hedging effectiveness. However, this figure corresponds to a 50 percent increase in hedging effectiveness relative to the ATM contract, and one-third if compared to the level reached at the optimal strike.

As observed, hedging a riskier commodity necessarily produces lower effectiveness, as the combined effect of basis and unhedgeable risk enlarges the mismatch between the exposure and the derivative position. Understanding to what extent the reduction of hedging effectiveness is a consequence of one or the other is captivating. Figure 2b disentangles the effect of basis and quantity risk, measuring the ES over the same exposure when one of the two sources of uncertainty is no longer present. Likewise, disentangling the exposure allows us to attribute the degree of moneyness of the put option contract to each of the two risks. To remove basis risk from our model we impose a unitary correlation between the spot and the asset underlying the option contract,[29] while the case of no-quantity risk is obtained setting the volatility of $Q_t$ equal to zero.

If we nullify one source of risk at a time, the ES of the exposure is reduced compared to the base case. The hedging effectiveness dramatically surges if we do not allow the model to include quantity risk (but only basis risk). At the optimal exercise price, the ES of the hedged position is 72 percent of the naked level, meaning a 28 percent risk reduction (relative to 12 percent of the combined risk case). When we assume that the hedger faces no proxy risk, but only uncertainty on the firm's output, the hedging effectiveness is only 15 percent, suggesting that production risk plays a major role.

As far as the derivation of the option optimal contract is concerned, we observe no important differences between the two risks, as the percentage of the spot position left unprotected appears similar in the two cases and nearly equal to the case of the combined risk exposure.

## 4.1 Comparative statics

[29] This also implies that $\theta_1 = \theta_2$, as we impose that spot and futures show the same stochastic behavior.

The conclusions drawn in the previous section reflect the parametrization of our model for the case of wheat. The scope of this section is extending the previous insights examining the role of each parameter through a sensitivity analysis. As before, the investigation will be carried out by presenting the impact over the level of hedging effectiveness and, most importantly, the contract optimality.

**Please insert Figure 3 about here**

Figure 3 presents the outcome of the sensitivity analysis run on the most relevant parameters. Figure 3A displays the ES for three different level of $\rho$, assumed as a proxy of basis risk. The solid line, that is the base case for wheat, is compared to two alternative situations where the correlation between the spot and the hedging position correlation is simulated to be lower. The ES upward shift associated with a decrease of $\rho$ is a natural consequence of the augmented risk. In terms of hedging effectiveness, a one basis point (0.01) correlation change yields a variation of approximately 0.27 percent on the goodness of the insurance strategy, confirming the relevance of the basis risk on option hedging.[30] Importantly, proxy risk also affects the contract optimality. In the base case, the optimal strike price is 0.88, indicating a 12 percent price exposure. When proxy risk increases ($\rho = 0.75$), the optimal exercise prices move towards a safer area, approaching the ATM level (0.92). Economically, this result is justifiable as the firm responds to the augmented price risk with a less OTM hedge. On the one hand, a less OTM hedge determines a smaller fraction of the quantity exposure which is protected. On the other hand, given the budget constraint, the firm is able to reduce the price uncertainty that has become the prevailing risk. In terms of a firm's policy, when basis risk becomes important, optimal option hedging foresees less OTM hedge. Hence, a 0.15 decrease in the spot-futures correlation yields a 0.04 shift in the optimal moneyness. The economic relevance of the optimality is significant, as the hedging effectiveness experiences a 2

[30] At the basis case optimal point the hedging effectiveness is 11.5 percent; a 0.15 $\rho$ reduction produces a 4 percent drop on the effectiveness level. Dividing the effectiveness change over the delta in correlation, we obtain 0.266 percent. A similar figure results by comparing the 0.75 to the 0.60 $\rho$ scenario.

percent ES decrease—approximately 29 percent of the overall hedging benefit—when the optimal contract is chosen instead of a naïve ATM hedge.

Figure 3B displays the effect of production risk ($\xi$) on the contract optimality and the hedging effectiveness. The resulting optimal strike price is only moderately affected by a change in quantity risk. A positive risk shift causes the optimal strike price to rise, indicating that production uncertainty moves the hedger towards a more protective option, although the change in the strike price is modest. Reducing quantity uncertainty affects the level of protection effectiveness—as expected—but also makes a proper choice of the optimal contract more relevant. This is due to the unhedgeable nature of the production risk, which makes it more difficult for firms to undertake effective insurance strategies based only on price protection.

By contrast, hedging cost plays a major role in determining the optimal strike price. Figure 3C shows the expected shortfall as a function of three levels of budget constraints. As for the other panels, the solid line indicates the base case (with $C$ equal to 1 percent of the spot position), while the dashed and dash-dotted lines display the 0.5 percent and 2.5 percent cases, respectively. Ahn *et al*. (1999) demonstrate, under a VaR approach, that the level of hedging expenditure does not impact on the optimal strike price. We find that the opposite is valid, as in our framework budget constraint exerts a great influence. When the hedging budget is halved, the optimal strike price is reduced from 0.88 to 0.84. With 2.5 percent budget constraint, the optimal contract goes almost ATM (to 0.97). As expected, a larger and less binding hedging constraint allows the firm to purchase more protection, leaving a smaller fraction of the price risk unhedged. The magnitude of the change in strike price is the resulting combined effect of the risk measure (the ES) and the inclusion of basis and production risk. We have already mentioned—within the milk (price risk) base case—that the strike price which minimizes the expected shortfall is a function of the hedging expenditure. However, when the two additional risk sources are included, the minimization solution becomes even more sensitive to the amount spent. Correspondingly, the economic relevance of choosing the optimal contract becomes noteworthy, as the difference between a naïve ATM contract and the optimal strike yields a 55 percent increase in hedging effectiveness when a 0.5 percent budget constraint is imposed.

The remaining part of Figure 3 illustrates the sensitivities to the model volatilities (panel D), hedging-period (panel E) and spot-quantity correlation (panel F).[31] Figure 3D analyzes the effect of the ratio $\sigma/\sigma'$ over the position's ES. The volatility ratio is negatively associated with the moneyness of the optimal contract, meaning that larger variability yields a lower strike price. An increase of the volatility of the spot position requires the hedger to enlarge the number of option contracts and—given the budget constraint—this translates into a lower moneyness. A similar—and likewise important—effect is notable in Figure 3E, where the negative relation between the optimal strike price and hedging horizon is displayed, suggesting that longer hedging periods are associated with less price protection. This is a consequence of the budget constraint, as any increase in $(T-t)$ raises the price of the put option and makes it more expensive for the hedger to effectively protect his exposure. Therefore, it is necessary to buy a cheaper contract so as not to leave an overly large fraction of the (quantity) exposure vulnerable to tail risk.

To conclude, Figure 3F illustrates the ES sensitivity to the spot-quantity correlation. Holding the level of production risk $(\xi)$ constant, a change in its correlation to spot price produces negligible effects on the contract optimality and the level of hedging effectiveness. Therefore, if spot-quantity correlation highly affects the functional shape of the risk exposure, justifying the option use for hedging purposes, it is only important to a limited extent for detecting the optimal price protection level.

### 4.2 A financial crisis scenario

As we previously showed, the choice of the optimal strike price is not as straightforward as we might expect. The tail risk reduction is substantial when the optimal contract is chosen in place of a naïve ATM hedge. Sensitivity analysis has also made clear that some parameters, more than others, play a significant role in determining the optimal exercise price, i.e. the budgeted hedging expense, the basis risk and the volatility ratio. While the first element is firm specific, the other parameters are exogenous, time-varying and subject to sharp variations when some market events take place. In this section, in order to illustrate the predictions of the hedging model in the case of a sudden change in the underlying parameters—such as

[31] Sensitivities to the diffusion drifts $(\mu, \mu', \mu_Q)$ and to the confidence level $(\alpha)$ are not reported here, as they exert no material influence on the optimal contract.

at the beginning of an economic downturn—we adjust the model parametrization and discuss the emerging insights. In terms of expected changes, a financial turmoil causes a volatility rise, both in terms of spot and futures, and a decline of prices. We therefore extend the case of wheat, estimating our model's parameters for the period 2008-2009 and generating a scenario analysis based on the following assumptions: 27.6 percent (42.0 percent) spot (futures) annual volatility, –24.0 percent (–20.3 percent) spot (futures) drift rate, spot-futures correlation drop (from 0.91 to 0.80).

The combined effect of this new parametrization is depicted in Figure 4. Panel A shows the expected shortfall of the hedged position measured in dollar terms (assuming a $100 million exposure). The option moneyness that minimizes the expected shortfall is about 0.74 (i.e., a strike price equal to 74 percent of the current wheat price), compared to 0.88 as before. Obviously, the increased volatility, rendering the option far more expensive, forces the firm to either reduce the fraction of the exposure to be secured or relax the amount of the price protection (choosing a lower option moneyness). The opportune choice of the strike price is not trivial, as the optimal option contract exhibits more than a $3 million expected shortfall reduction if compared to the ATM hedge. The hedging effectiveness is poorer with respect to the no-crisis scenario, as intuition would suggest. The combined effect of higher volatility and basis risk severely increases tail risk.

Panel B shows, instead, the expected shortfall of the hedged position as a percentage of the naked position expected shortfall. The inspection of this figure strengthens our conclusions, as we note that the choice of a sub-optimal exercise price, such as the ATM put option, yields a 6 percent reduction of the unhedged position expected shortfall, *vis-à-vis* a reduction of almost 13 percent when the optimal moneyness is chosen. This result suggests that the tail loss would be twice as large without properly taking into account the optimal hedging policy.

## 5. Conclusions

The soaring volatility in the financial markets experienced over the last period makes the role of corporate risk management even more crucial than in the past. Once it has decided to protect against market risks, any firm has to choose whether to hedge using linear (e.g., forwards, futures and swaps) or non-linear instruments (e.g., options). The optimality of one strategy relative to the other has been widely discussed among scholars, and so has the derivation of the optimal hedge when linear instruments are used. By contrast, the optimal hedging policy with option contracts has been almost overlooked, despite this topic raises non-trivial theoretical issues along with a potential genuine interest from hedging firms.

In this paper we propose a model to determine the optimal option hedging strategy for a firm facing not only price risk—as is common in the literature—but also explicitly considering the role of production and basis risk. We argue that neither of these two risks can be neglected. Production risk has been documented as being the main rationale for using options for hedging purposes. Basis (proxy) risk is instead deeply rooted in any hedging strategy involving non tailor-made derivatives. Both these risks are incorporated into our model, which yields the optimal hedging strategy when a measure of tail risk, i.e. expected shortfall, is considered.

In contrast to prior findings, we show that the optimal option contract is greatly influenced by the amount of cash spent on the hedging program. This result holds also within a framework of no quantity and basis risk. Besides, we investigate the separate effect on the optimal hedging strategy of quantity and basis risk. When only the former is in place, the hedging effectiveness notably decreases, confirming the importance of production risk. The latter materially affects the choice of the optimal contract, as a one basis point (0.01) change in correlation between the spot position and the option underlying asset yields an approximate 0.27 percent variation in hedging effectiveness. An increased basis risk pushes the hedger toward a less OTM option, thus leaving a smaller fraction of its price risk unexposed

To illustrate the predictions of our model in the case of a sudden change in the underlying parameters we simulate a financial crisis scenario. The increased volatility pushes the price of the put option upward and forces the firm to either reduce the fraction of the exposure to be secured or to relax the amount of the price protection, choosing a lower option moneyness. The choice of a sub-optimal exercise price, such as a naïve ATM hedging strategy, produces an expected shortfall reduction of only half the level obtained when the optimal moneyness is chosen. This result confirms the economic relevance of choosing the optimal hedging policy.

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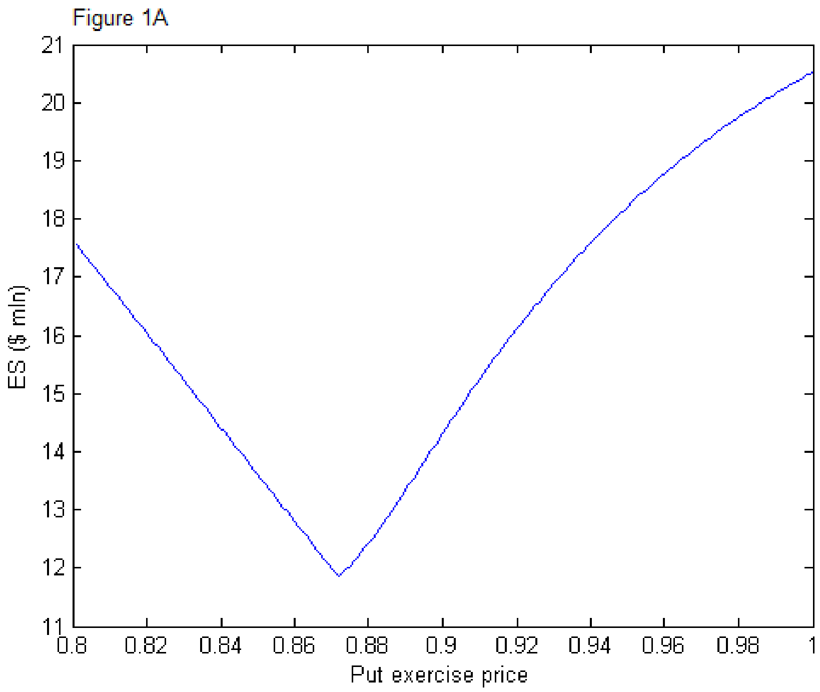
**Table 1. – Descriptive statistics.**

This table reports the descriptive statistics and correlations for a sample of widespread agricultural and food commodities. Annual prices and production data are from the US Department of Agriculture (USDA), National Agricultural Statistics Service (NASS), while annual futures prices are from Thomson Reuters Datastream. Reported values are estimated in the window period 1995-2010. All statistics except correlations are expressed in percentages.

| Commodity | Type | Mean | Median | SD | 1Q | 3Q | Correlations | | |
|---|---|---|---|---|---|---|---|---|---|
| Cattle | Spot | 1.43 | 5.10 | 12.52 | -2.94 | 8.91 | 1.00 | | |
| | Futures | 3.41 | 2.23 | 7.87 | -1.02 | 7.80 | 0.29 | 1.00 | |
| | Quantity | -0.14 | -0.17 | 1.48 | -0.99 | 1.03 | 0.13 | 0.13 | 1.00 |
| Corn | Spot | 4.06 | -0.66 | 20.10 | -12.16 | 17.16 | 1.00 | | |
| | Futures | 5.57 | 4.96 | 24.14 | -12.96 | 24.65 | 0.47 | 1.00 | |
| | Quantity | 3.21 | -0.50 | 10.00 | -5.21 | 9.86 | -0.27 | 0.17 | 1.00 |
| Cotton | Spot | 1.45 | 3.56 | 25.25 | -15.18 | 25.15 | 1.00 | | |
| | Futures | 2.30 | -5.04 | 22.53 | -12.90 | 10.93 | 0.54 | 1.00 | |
| | Quantity | -0.87 | 0.25 | 20.31 | -13.37 | 11.28 | -0.13 | -0.02 | 1.00 |
| Hogs | Spot | 3.10 | 2.01 | 21.40 | -9.79 | 21.80 | 1.00 | | |
| | Futures | 4.59 | 2.48 | 18.89 | -5.18 | 20.82 | 0.91 | 1.00 | |
| | Quantity | 1.49 | 1.59 | 3.25 | -0.10 | 3.37 | -0.57 | -0.55 | 1.00 |
| Milk | Spot | 2.88 | -0.52 | 20.39 | -12.36 | 20.35 | 1.00 | | |
| | Futures | 1.03 | -1.24 | 20.26 | -17.40 | 21.15 | 0.98 | 1.00 | |
| | Quantity | 1.46 | 1.79 | 1.50 | 0.23 | 2.76 | -0.27 | -0.20 | 1.00 |
| Rice | Spot | 2.75 | 0.81 | 24.02 | -11.15 | 18.13 | 1.00 | | |
| | Futures | 7.71 | 18.62 | 28.03 | -23.26 | 29.30 | 0.75 | 1.00 | |
| | Quantity | 0.39 | 1.36 | 10.78 | -4.79 | 8.83 | -0.53 | -0.27 | 1.00 |
| Soybeans | Spot | 3.47 | -1.35 | 18.79 | -5.08 | 14.58 | 1.00 | | |
| | Futures | 4.93 | 2.40 | 20.61 | -14.27 | 21.40 | 0.47 | 1.00 | |
| | Quantity | 2.13 | 2.89 | 10.39 | -4.00 | 9.67 | -0.73 | -0.15 | 1.00 |
| Wheat | Spot | 3.10 | 4.53 | 20.67 | -6.63 | 21.96 | 1.00 | | |
| | Futures | 3.38 | 5.20 | 21.07 | -9.60 | 19.29 | 0.91 | 1.00 | |
| | Quantity | -0.55 | -2.75 | 14.81 | -11.17 | 6.41 | -0.12 | -0.12 | 1.00 |

**Figure 1. – Expected shortfall for a milk producer**

This figure shows the expected shortfall of the hedged position (in USD million—top chart, Figure A—and in percentage of the unhedged expected shortfall—bottom chart, Figure B) as a function of the put option moneyness (defined as the ratio between the strike price and the spot price). The exposure refers to a milk producer with a $100 million long position. The confidence level is set at 1 percent. The parametrization of the model follows milk statistics depicted in Table 1.



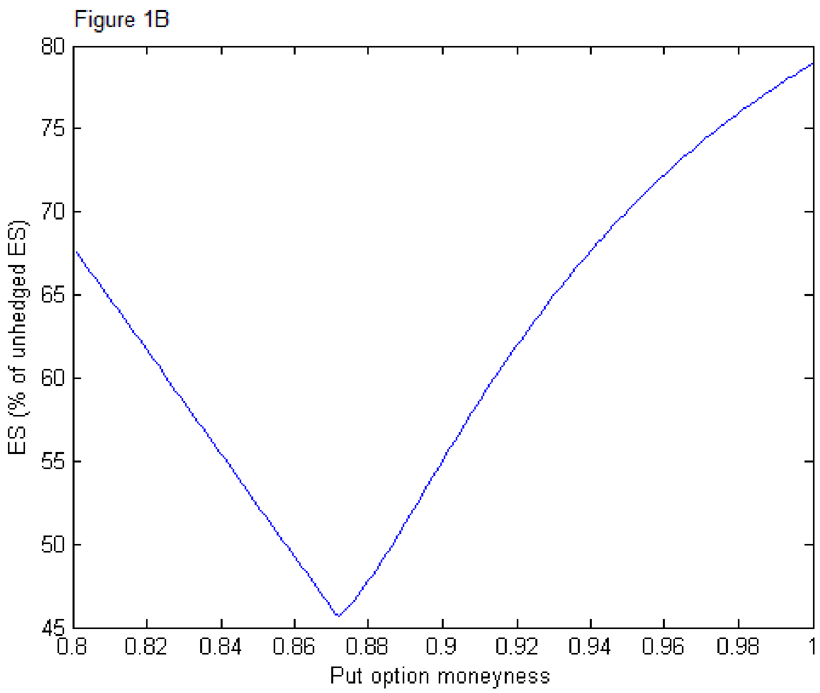
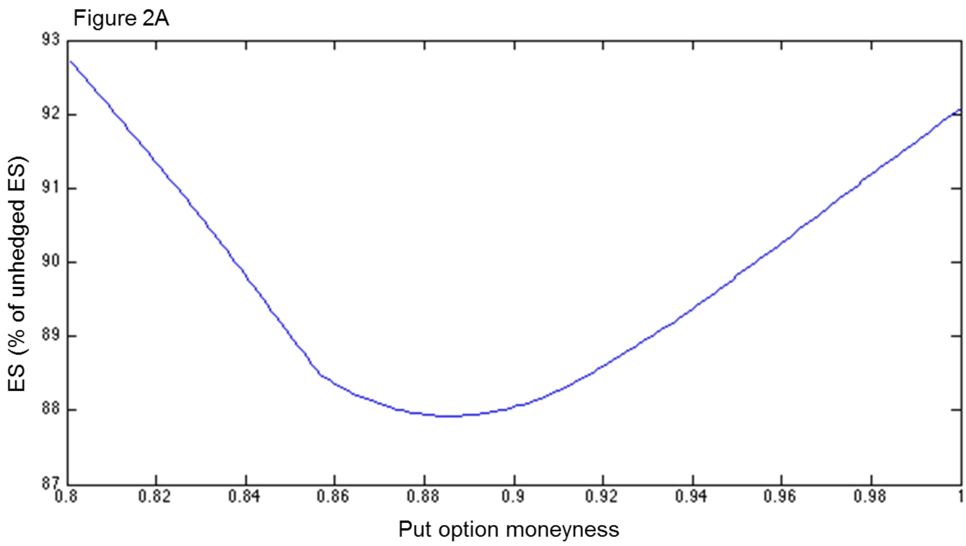



**Figure 2. – Disentangling basis and production risk.**

The upper chart (Figure A) shows the expected shortfall of the hedged position (as a percentage of the unhedged expected shortfall), as a function of the put option moneyness, i.e. the ratio between the strike and the spot price. The exposure refers to a wheat producer with a \$100 million long position. The confidence level is set at 1 percent. The parametrization of the model follows the wheat statistics depicted in Table 1. The lower chart (Figure B) disentangles the effects of basis and production risk for the same wheat producer. No price (production) risk is obtained setting $\rho = 1$ and $\theta_1 = \theta_2$ ($\xi = 0$).



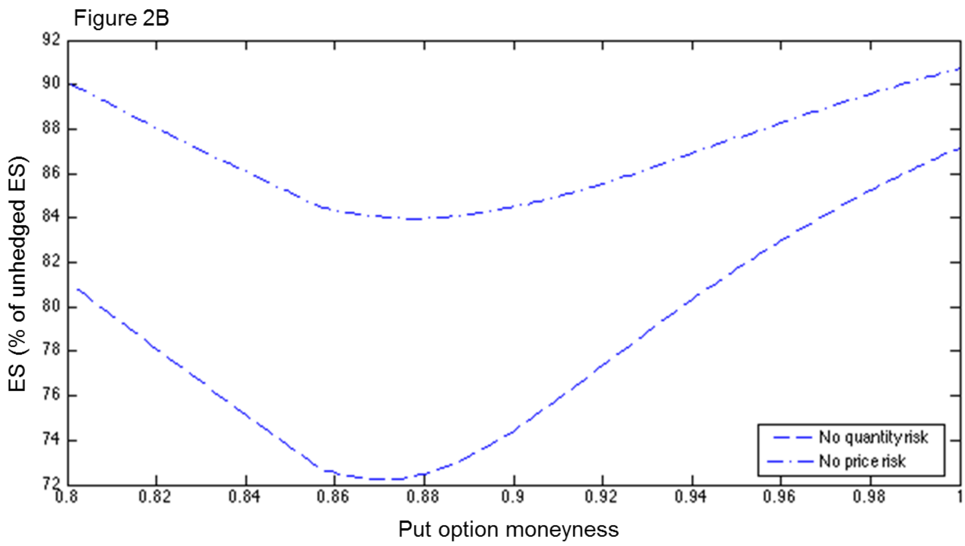



**Figure 3. – Sensitivity analysis**

This figure shows the effects of a change in the relevant parameters on the expected shortfall of the hedged position (as a percentage of the unhedged expected shortfall) as a function of the put option moneyness, i.e. the ratio between the strike and the spot price. The exposure refers to a wheat producer with a \$100 million long position. The confidence level is set at 1 percent. The parametrization of the model follows the wheat statistics depicted in Table 1 The simulated parameters are: the correlation between the spot position and the asset underlying the option position, $\rho$ (figure A), the production volatility, $\xi$ (figure B), the hedging cost, $C$ (figure C), the ratio between the volatility of the spot position and the volatility of the underlying the option position, $\sigma/\sigma'$ (figure D), the hedging horizon, $(T-t)$ (figure E) and the ratio between the quantity-spot price correlation and quantity-futures price correlation, $\theta_1/\theta_2$ (figure F). The solid line shows the base case, while the dash and dash-dotted lines show the simulated changes in the parameters.

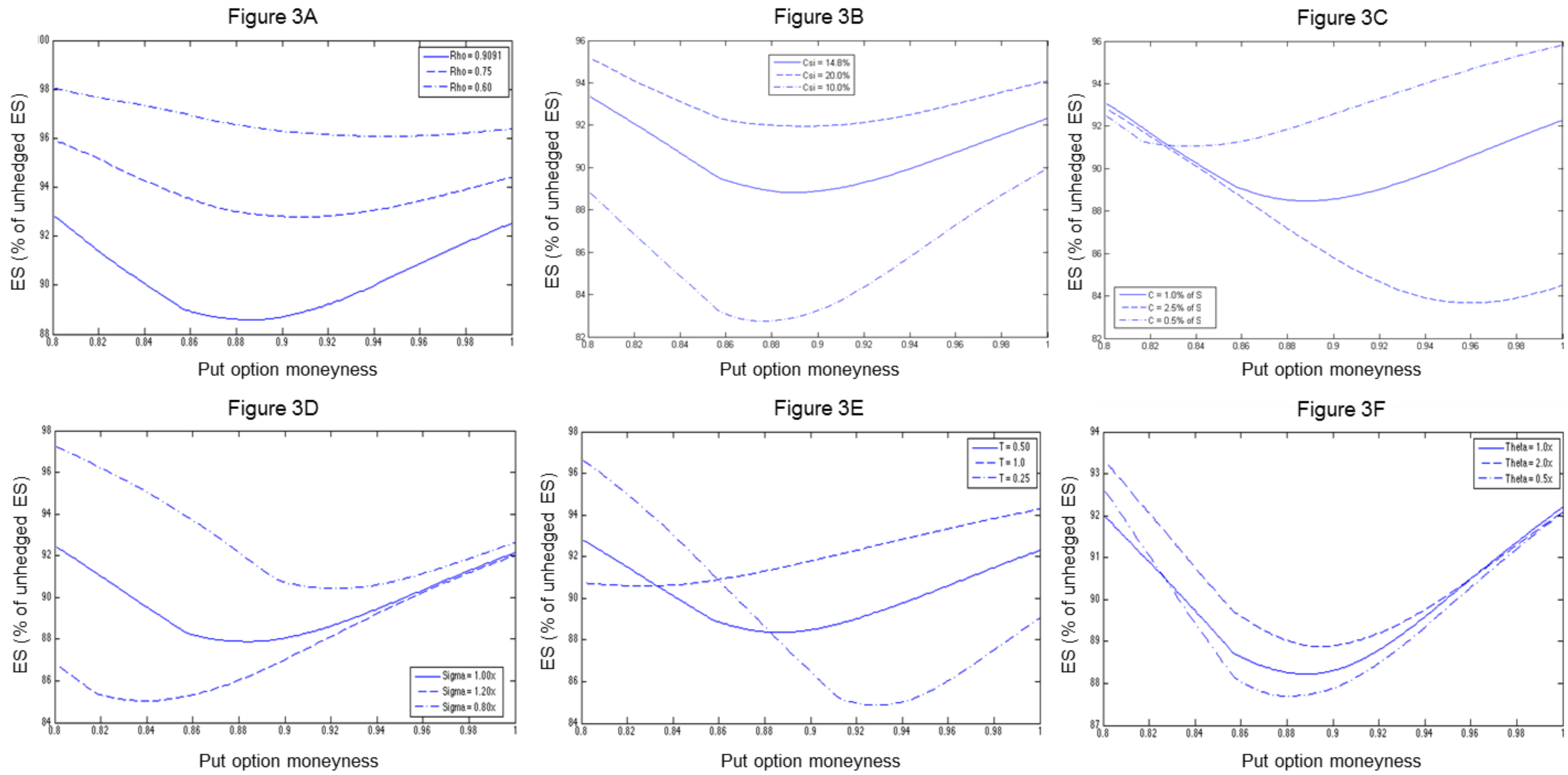


**Figure 4. – A crisis scenario**

This figure shows the effect of a combined change in the relevant parameters on the expected shortfall of the hedged position (in USD million—top chart, Figure A—and in percentage of the unhedged expected shortfall—bottom chart, Figure B) for a wheat producer as a function of the put option moneyness, i.e. the ratio between the strike price and the spot price. The parametrization of the base scenario (solid line) is as in figure 2. The change in relevant parameters (dashed line) reflects a turbulent market scenario (e.g., the financial crisis) in which both spot and futures volatility rise, both spot and futures drifts fall, and the spot-futures correlation diminishes. Estimating data for the period 2008-2009 yields $\sigma = 27.6$ percent, $\sigma' = 42.0$ percent, $\mu = -24.0$ percent, $\mu' = -20.3$ percent, $\rho = 0.80$.