Collateralised Short-term Debt and Self-fulfilling Fire Sale

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Abstract

This paper shows how a crisis equilibrium can arise endogenously when financial firms rely on collateralised short-term borrowing. Due to a novel feedback mechanism between firms' ex-ante risk-taking incentives and ex-post fire sale discount of the collateral, multiple equilibria exist in a self-fulfilling fashion, resulting in discontinuous jumps in equilibrium amount of credit rationed, firms' default risks, debt yields, and collateral asset discount. These results suggest such a financial system is inherently fragile and social planner can improve welfare by asset price guarantee to eliminate pessimistic expectation and by direct collateral market support to sustain the constrained efficient equilibrium.

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1 Introduction

In the modern financial intermediation system, financial institutions (for simplicity I call them 'banks' in the paper) typically collateralise their asset-in-place to raise short-term funding in the form of repurchase agreement (repo) and Asset-backed Commercial Paper (ABCP) to finance longer term illiquid investment[1]. The collateral assets are usually relatively safe asset with little credit risk such as AAA rated sovereign debt and Asset-Backed-Securities. Nonetheless, as seen in the Global Financial Crisis 2007-2009 and the on-going European sovereign debt turmoil, these collateral assets could become *illiquid* in the sense that the market value of these asset can be substantially lower than their fundamental, hold-to-maturity value. This wedge, also known as 'fire-sale discount', can arise if the marginal buyer or the highest valuation holder of the asset in the economy has limited capital or has to sell the asset to some lower valuation buyers for his own liquidity reason[2]. Importantly, the equilibrium fire-sale discount can both affect *and* be affected by the investment decision of agents in the markets. This paper explores this inter-dependency between ex-ante agents' investment decisions and ex-post fire-sale discount, highlights the fragility of the financial system in the presence of this feedback effect, and discusses the corresponding policy implications.

The main contribution of the paper is to show that a financial system which relies on collateralised short-term borrowing is *inherently fragile*, i.e. a systemic crisis can occur endogenously even in the absence of aggregate shock. This inherent fragility is caused by a novel feedback mechanism between banks' ex-ante risk-taking incentives and ex-post fire-sale discount in the collateral market. When the collateral market can potentially be illiquid, for instance when the potential buyer of the collateral has limited capital, there exists *multiple, rational expectation equilibria* which can be *Pareto-ranked*. The Pareto-dominated equilibrium is caused by the pessimistic expectation of the lenders and has features resemble a financial crisis such as sharp increase in debt yields, significant distress

[1] Adrian and Shin (2011) called this 'Market-based financial system'

[2] For a formal discussion of the concept of 'fire-sale' and a comprehensive survey of its application in the Macroeconomics and Finance literature, see Shleifer and Vishny (2011).

in asset market, and substantial credit rationing, etc. In other words, a crisis can arise in a *self-fulfilling* fashion and the switching from one equilibrium to another will lead to dramatic movement in financial markets, consistent with observations in recent financial turmoil. Finally, the self-fulfilling nature of the inefficiency arising from the illiquidity in the collateral asset market suggests that central bank can improve welfare by policy like asset price guarantee to eliminate the lenders' pessimistic expectation and by direct lending or injecting a critical amount of liquidity to restore the constrained efficient equilibrium. In contrast to the well-understood moral hazard critique of government intervention in asset market, this paper suggests that by ex-ante committing to intervene in collateral market central bank can *reduce* moral hazard problem and promote financial stability.

To be more specific, I model a continuum of banks raising finance from their cash-rich lenders in the form of collateralised short-term debt to financial profitable, long-term investment. At the interim, lenders will observe the solvency of the banks and in the case of insolvency they will liquidate the collateral in the market. There are two key frictions in the model: i) after raising finance, banks can privately choose the riskiness of the investment project and, ii) there is illiquidity in the collateral asset market, modelled as a liquidation value function decreasing in the measure of collateral asset being liquidated.

As mentioned above, the mechanism driving the financial fragility is a *self-reinforcing, feedback loop* between banks' ex-ante risk-taking incentives and the ex-post fire-sale discount of the collateral asset: When creditors expect that they have to liquidate the asset in an illiquid market with substantial discount should the bank they financed becomes insolvent, they demand a higher face value of the debt (or equivalently, debt yields) to break-even. Since the downside is protected by limited liability, bank facing a higher debt yields will have stronger incentive to engage in risk-taking. Hence the *ex-post illiquidity of collateral asset market induces banks to take risk* via the standard asset-substitution channel as in Jensen and Meckling (1976). On the other hand, the higher (idiosyncratic) credit risk chosen by banks will lead to more insolvency and distressed asset sales ex-post. Thus in aggregate, *banks' individual decision to take risk depresses equilibrium asset liquidation discount.*

Under rational expectation, two equilibria can arise. When creditors are 'optimistic' in the sense that they anticipate a small liquidation discount for the collateral asset, they charge a lower yields and hence banks take less risk. In aggregate there will be fewer banks become insolvent and thus fewer assets will be liquidated in the market, confirming the anticipated small liquidation discount. Another *fire-sale* equilibrium can arise in the same self-fulfilling rational expectation fashion when the creditors are 'pessimistic'. Figure 1 summarises the feedback mechanism behind the self-fulfilling crisis equilibrium.

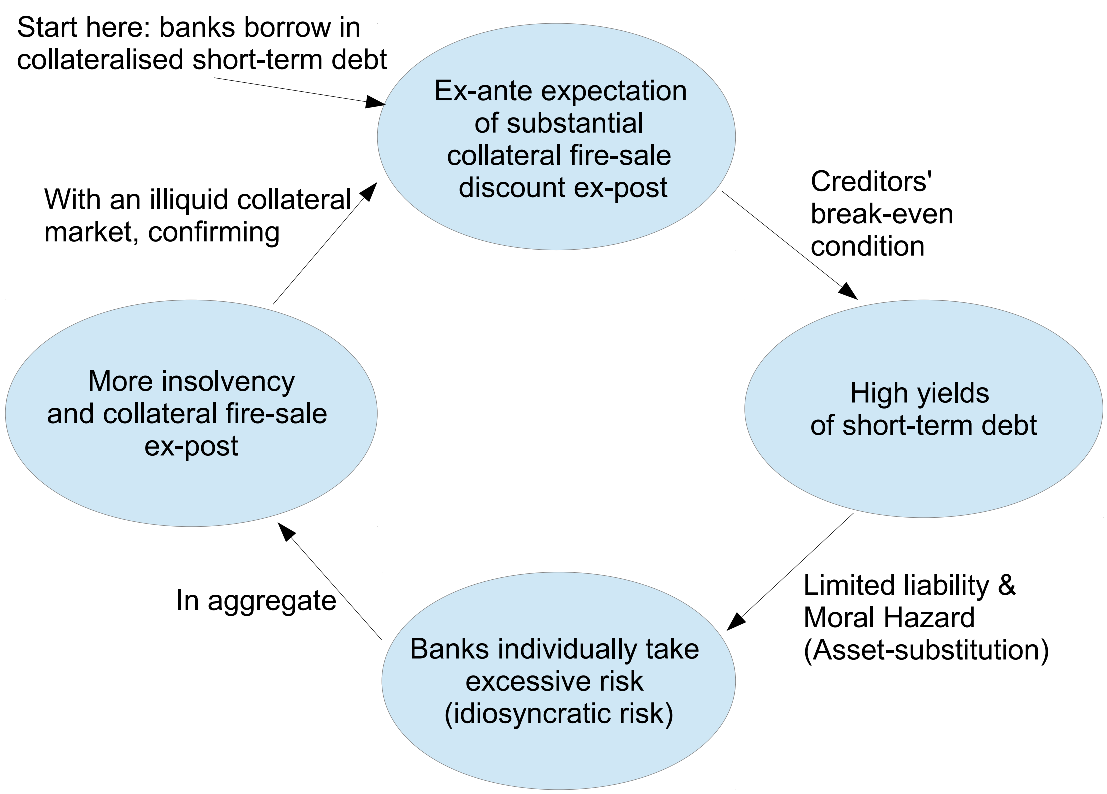

Figure 1: Self-fulfilling fire-sale when the collateral asset market is sufficiently illiquid

This simple yet powerful feedback mechanism can explain the sudden dry-up in the credit market, the discontinuous jumps in banks' default risk and short-term debt yields, and the collapse of asset-backed securities and peripheral Euro-area sovereign debt prices, as seen in the Global Financial Crisis 2007-2009 and the recent European Sovereign Debt turmoil. These changes comes from the switching of equilibria, either due to the shift between 'pessimistic' and 'optimistic' regimes of the agents when the market is moderately illiquid, or when the market illiquidity approaches a critical level that a small increase in market illiquidity will cause the equilibrium to switch to the unique crisis equilibrium.

This multiple equilibria result is reminiscent of the panic-based bank runs model of the seminal paper by Diamond and Dybvig (1983). Creditors in each bank fail to coordinate their belief about the degree of market distress ex-post and their collective 'pessimism' leads to the existence of inefficient equilibrium. A creditor's pessimistic belief indirectly exerts a negative externality on others via the *moral hazard channel* and the *illiquid collateral asset market*. When one believes a deep discount fire-sale will arise, her demand of higher yields induces her bank to take higher risk, (marginally) steepening the expected equilibrium liquidation discount of the collateral received by every creditor in the case of insolvency.

The Pareto-inefficiency stems from friction in the collateral market and naturally creates potential role for central bank to intervene in the market. In particular, central bank can improve welfare by acting as the 'market maker of last resort', a term coined by Willem Buiter (see Buiter and Sibert (2007)), who directly intervenes in some key asset markets to ensure a proper functioning of the financial intermediation system. I show that when the collateral market is moderately illiquid, central banks can eliminate agents' pessimistic beliefs and hence rule out the inefficient crisis equilibrium by simply guaranteeing a lower bound of collateral asset prices, *without* any direct intervention in the market. When the market becomes so illiquid that the unique equilibrium is the crisis equilibrium, central banks can act as a modern version of 'lender of last resorts' and provide the critical amount of liquidity to the market via various programs such as direct lending against illiquid collateral, swapping illiquid collateral with liquid treasuries, direct asset purchase to support price, etc. In short, this paper provides economic rationale for central banks' unconventional intervention in some key asset markets such as the ABCP and ABS markets during the Global Financial Crisis 2007-2009.

Related Literature (Incomplete) The negative feedback mechanism in this paper is similar to the one in Brunnermeier and Pedersen (2009) and Danielsson, Shin, and Zigrand (2011) which show that when financial institutions operate under a Value-at-Risk constraint, there exists a feedback loop between the financiers' perceived asset price volatility and the banks' asset liquidation decisions: a high expected price volatility tightens the VaR constraint, forcing liquidation of asset in an illiquid market, resulting in large fluctuations in asset price.[3] These papers focus on how exogenous shocks like liquidity demand shocks are amplified due to the above exogenous constraints, there is no uncertainty in my model and the risk is coming from moral hazard concern of individual agents and arises in a self-fulfilling fashion. I also focus on the incentive distortion effects of debt, without imposing exogenous margin and VaR constraints.

Inefficiency stemming from self-fulfilling coordination failure in the context of recent crisis can also be found in Malherbe (2012) and Bebchuk and Goldstein (2011). Malherbe (2012) shows a self-fulfilling liquidity dry-up phenomenon in a situation where a bank may need to liquidate its asset in a lemons market in the next period due to some liquidity shock to its depositor. If every banks anticipate a severe lemon problem at the interim, each of them will hoard liquidity, resulting in fewer liquidity-driven sales of average quality asset in the market, leading to a severe lemons problem. In contrast to adverse selection, I study how moral hazard (asset-substitution) can arise in a self-fulfilling manner. By assuming bank lending to the corporate sector is strategic complement, Bebchuk and Goldstein (2011) shows that credit squeeze can occur as a coordination failure. By contrast in this paper, I show strategic complementarity between banks' risk-taking decisions arises endogenously from the illiquidity of asset market.

This paper also relates to the literature on the consequence of fire-sale and its associated

[3] See also Gromb and Vayanos (2002) and Vayanos (2004) for models with limits to arbitrage due to margin and agency constraint

inefficiency. Diamond and Rajan (2011) shows that anticipation of fire-sale of assets can lead to credit crunch. The reason is that in order to participate in the profitable purchase of fire-sale, potential buyers will hoard cash and forgo productive real investment ex-ante, thus reducing the supply of credit in the economy. I show that the anticipation of fire-sale could not only result in inefficient investment, but also lead to fire-sale itself endogenously. Stein (2012) assumes a 'money-like' premium in lenders' preferences and shows that banks tend to create too much safe asset by excessive short-term borrowing. The private money creation equilibrium is constrained inefficient because banks fail to take into account of the fire-sale pecuniary externality exerted on others when they become insolvent. I show banks would engage in risk-taking due to the interaction between a similar pecuniary externality and the moral hazard constraint, leading to inefficiency and excessive risk in the system, without imposing a money-like premium on lenders' preferences.

2 Model

Banks and Projects Consider a three-dates ($t = 0, 1, 2$) model with a continuum of banks in a banking sector. Riskfree interest rate is normalised to zero and all agenets are risk-neutral. Banks are identical ex-ante and have a unit of asset-in-place with no cash and debt. At $t = 0$ each bank has the opportunity to invest in one of the following two projects:

1. 'Efficient' or 'Safe' project which produces cashflow X_{p_1} at $t = 2$ with probability p_1 and 0 otherwise

2. 'Inefficient' or 'Risky' project which produces cashflow X_{p_2} at $t = 2$ with probability p_2 and 0 otherwise

Both of these investment projects require initial investment \$1. The primitives are set such that $p_1 X_{p_1} > p_2 X_{p_2} \geq 1$, $X_{p_1} < X_{p_2}$ and hence $p_2 < p_1$, meaning both projects have positive Net Present Value and while the safe projects offer a higher NPV, the risky project produces a higher cashflow given success. Project realisations are independent across banks. After securing finance, bank managers can privately decide which project to invest. In line with the classic asset substitutions models such as Jensen and Meckling (1976), I assume the project cashflow is not verifiable and thus repayment cannot be contingent on the realisation of X.[4] One can regard the projects as commercial lending to businesses and because commercial lending involves banking expertise and soft information, the project choice is hardly observable to outsiders and difficult to distinguish efficiently risky from excessively risky projects even ex-post. The success or failure of the projects is known at $t = 1$ and the corresponding cashflow is realised and distributed at $t = 2$.

Collateral Asset and Financing Besides the investment opportunity, each bank has an asset-in-place which can be used as a collateral for borrowing. The collateral asset pays a random terminating dividend $\tilde{v} \in [0, \bar{v}]$ with mean v at $t = 2$. To focus on the effect of illiquidity in the collateral asset market, rather than the credit risk of the collateral asset, I assume the dividend and projects realisations are uncorrelated and there is no aggregate shocks to the collateral asset, i.e. the expected terminating dividend will always equal v at both $t = 0$ and $t = 1$. I also assume banks can only borrow in the form collateralised short-term debt. A bank borrows \$1 from creditor and promises to repay r at $t = 1$ and if the bank fails to repay, the creditor automatically owns the collateral. Since neither of the projects produce cashflow at the interim, should the creditors decide not to roll-over the debt, the bank will be forced to liquidation and the creditors will seize the collateral asset[5].

Creditors and rollover decision Each bank is matched with a representative creditor who has cash \$1 to lend. After the initial financing, at $t = 1$ creditors will receive a non-contractible signal about their bank's project has succeeded or not, that is, whether X or 0 has realised. In order to isolate the effect of noisy, wrongful liquidation on inefficiency, throughout the paper I assume the signal is perfect and hence the creditors essentially

[4] while success or failure of the project is verifiable,i.e. the court can tell X from 0 but not X_{p_1} from X_{p_2}

[5] As it will be clear soon, hoarding liquidity to meet withdrawal demand has no role in this paper because there is no liquidity shock to creditors. Withdrawal is completely based on information about the solvency of the bank

observe the solvency of their banks. If projects have succeeded, creditors know they will be repaid for sure at $t = 2$ so they will roll over their short-term debt. One can think of a solvent bank issuing riskless (in the eye of its own creditors) securities backed by the succeeded project which is worth r to repay its creditor at $t = 1$. On the other hand, when the project fails the bank can no longer repay its debt, creditors will seize the collateral asset and sell it in the market at $t = 1$.

Endogenous liquidation value The final key element in the model is the illiquidity in the collateral market at $t = 1$. When a creditor decide to sell the collateral asset in the market, she will take the liquidation value βv as given (due to her infinitesimal size) and the equilibrium liquidation value is decreasing in both the aggregate amount of the collateral sold and the degree of illiquidity $\alpha \in [0, +\infty)$ in the market. Hence the equilibrium liquidation value function is a downward-sloping curve in the amount of collateral sold and α controls the absolute size of the slope, i.e. price impact. More discussion on the specific structure and micro-foundation of the equilibrium liquidation discount function in section 2.2. It is important to emphasise that the degree of illiquidity α is an exogenous parameter and common knowledge. Hence the market illiquidity is *not* a source of aggregate risk and in fact there is no aggregate uncertainty in the model.

Most of the analysis of the paper will be around the effect and the determination of the *endogenous liquidation discount factor* β. Please note that the conventional notion of fire-sale discount should correspond to $(1 - \beta)$.

Before we turn to the individual bank-creditor contracting problem, the following timeline summarises the sequence of events:

t=0

- A continuum of bank each needs to borrow \$1 to invest, using an asset-in-place worth v as collateral
- The degree of illiquidity in the collateral market α is publicly observed
- Each bank offers a collateralised short-term debt contract to its creditor with promised payment r
- Bank chooses project (p_1 or p_2) privately after financing
- Projects and dividend of collateral realisations are independent

t=1

- Projects realised. With perfect signals, creditor knows whether her bank's project has succeeded or failed
- Creditors in solvent banks will roll over their debt and receive r at t=2
- Creditors in insolvent banks will seize and sell the collateral asset in the market, taking the endogenous expected liquidation value βv as given
- βv the liquidation value is determined in equilibrium by the amount of asset being liquidated and the degree of illiquidity α

t=2

- Collateral asset's dividend realised
- Succeeded projects' cashflow matures
- Creditors in succeeded banks receive repayment r, and the banks keep all the remaining cashflow

2.1 Bank-Creditor Contracting problem

We first study the contracting problem between a bank and its creditor. Each bank offers a collateralised short-term debt contract to its creditor. More specifically, bank promises to repay a r (or gross debt yields r) should its creditor demands at $t = 1$. If the creditor demands a repayment at $t = 1$ (i.e. not rolling over the debt) and the bank fails to fulfil the demand, creditor can liquidate the bank and its project and seize the collateral asset.

Hence the bank's project choice decision is as follows:

Banks' investment decision By offering a short-term debt with yields r, bank will choose the safe project over the risky one if and only if the following incentive compatibility constraint is satisfied:

$$p_1(X_{p_1} + v - r) \geq p_2(X_{p_2} + v - r) \tag{1}$$

The left hand side is the expected payoff to the bank if the safe project is chosen. With probability p_1, the project succeeds and the total payoff to the bank is the cashflow from project and the expected maturing dividend of the collateral asset less the promised repayment to creditor. Bank will be liquidated and get zero payoff if the project fails. Comparing with the expected payoff from choosing the risky project, the above incentive compatibility constraint implies a standard risk-taking condition: bank will choose the safe project if the yields is less than a threshold r_{AS} (shorthand for asset substitution)

$$r \leq \frac{(p_1 X_{p_1} - p_2 X_{p_2})}{p_1 - p_2} + v \equiv r_{AS} \tag{2}$$

Gross yields of debt The gross yeilds of debt is determined by the break-even constraint of the creditor. For a given expected liquidation discount factor $\beta \in [0, 1]$ of the collateral asset in the case of bank defaults, the break-even condition is:

$$\hat{p}r + (1 - \hat{p})\beta v \geq 1 \tag{3}$$

where $\hat{p}$ is investor's conjectured project choice as she cannot observe the project choice of bank directly. From bank's incentive compatibility constraint, however, creditor can rationally anticipate the bank's project choice by looking at the proposed gross yields of debt r, i.e. whether $r > r_{AS}$ or not. Thus the conjectured project choice $\hat{p}$ will always be correct in equilibrium.

Formally, bank will solve the following optimization problem:

$$\max_r \quad p_i[X_{p_i} + v - r] \qquad \text{(Bank's expected payoff)}$$

$$\text{subject to } p_i = \begin{cases} p_2 & \text{for} \quad r > r_{AS} \\ p_1 & \text{otherwise} \end{cases} \qquad \text{(Bank's IC)}$$

$$\hat{p}r + (1-\hat{p})\beta v \geq 1 \qquad \text{(Creditor's PC)}$$

$$\hat{p} = p_i \qquad \text{(Creditor's RE)}$$

$$\pi(p_i, v) \equiv max_r \quad p_i[X_{p_i} + v - r] \geq v \qquad \text{(Bank's PC)}$$

In words, the bank manager will offer a contract with gross yields r to maximize profit subject to incentive compatibility constraint (IC), creditor's participation (PC) and rational expectation constraints (RE). If the last constraint, the bank's participation constraint, is not satisfied, then the bank will not take any project or choose to be credit-rationed. Notice that the key variable of interest is the collateral's expected liquidation discount factor β and the following lemma will first characterize the optimal short-term debt contract, assuming the bank's participation constraint is satisfied.

Lemma 1 *Assuming the bank's participation constraint is satisfied, there exists an β_{AS} (shorthand for 'asset-substitution') such that*

1. *for $\beta < \beta_{AS}$, bank will offer a collateralised short-term debt contract with a yields $r(p_2, \beta)$ and choose the risky project.*

2. *for $\beta \geq \beta_{AS}$, bank will offer a collateralised short-term debt contract with yields $r(p_1, \beta)$ and choose the safe project.*

where

$$r(p_i, \beta) = \frac{1-(1-p_i)\beta v}{p_i} \quad \textit{for} \quad i = \{1, 2\} \quad \textit{and} \tag{4}$$

$$\beta_{AS} = \frac{1-p_1 r_{AS}}{(1-p_1)v} \tag{5}$$

Proof: *See Appendix.*

Lemma 1 demonstrates how the expected *ex-post* liquidation value of collateral asset affects *ex-ante* risk-taking behaviour of bank, which is the first key channel of instability in this paper. In fact, the equilibrium project choice can be written as a function of β

$$p(\beta) = \begin{cases} p_2 & \text{for} \quad \beta < \beta_{AS} \\ p_1 & \text{otherwise} \end{cases} \tag{6}$$

The intuition of the result is as follows: Creditor who needs to break-even requires a high yields when the expected liquidation discount factor is low. With a high enough yields of debt, however, bank *cannot commit* to choose the safe project and hence in equilibrium bank will take the risky project. To see this, suppose the expected liquidation discount factor β is equal to the threshold β_{AS}. Using the definition of β_{AS}:

$$p_1 r_{AS} + (1 - p_1)\beta_{AS} v = 1$$

At this equilibrium, bank will choose the safe project and offer a yields $r(p_1, \beta_{AS}) = r_{AS}$. Now suppose the expected liquidation discount factor is below β_{AS}. In order to satisfy the break-even constraint, the required yields r has to be higher than r_{AS}. But this violates bank's incentive compatibility constraint and hence contract with safe project is not feasible.

With lemma 1, now we can study the bank's participation constraint. Plugging the equilibrium yields of debt $r(p(\beta), \beta)$ in the bank's participation constraint, we have

$$p(\beta)X_{p(\beta)} - 1 \geq (1 - p(\beta))(1 - \beta)v$$

The above condition implies bank will only take the project if and only if the Net Present Value (NPV) of the project is higher than expected cost of fire-sale. To make both the asset-substitution and credit rationing problem non-trivial, I will make the following parameters restrictions:

Assumption 1 *:*

(Both safe and risky could be taken in equilibrium) $\beta_{AS} \in (\beta_{CR}, 1)$

(Credit rationing can happen) $\beta_{CR} := 1 - \dfrac{p_2 X_{p_2} - 1}{(1 - p_2)v} > 0$

With the above assumption, I can fully characterise the individual bank-creditor contracting problem

Proposition 1 *(The bank's optimal investment strategy) Taking the expected liquidation discount factor of the collateral β as given, bank will do one of the following:*

1. *for $\beta \in [0, \beta_{CR})$, the bank will choose to forgo the project. (Credit Rationing)*

2. *for $\beta = \beta_{CR}$, the bank will take the risky project with probability λ and forgo the project with probability $(1 - \lambda)$, as the bank is indifferent between taking the risky project and forgoing it.*

3. *for $\beta \in (\beta_{CR}, \beta_{AS})$, the bank will take the risky project.*

4. *for $\beta \in [\beta_{AS}, 1]$, the bank will take the safe project.*

when assumption 1 *holds. When bank decides to invest, it will offer a take-it-or-leave it collateralised short-term debt contract as described in lemma 1.*
Proof: *See the above discussion.*

Proposition 1 can also be summarised graphically:

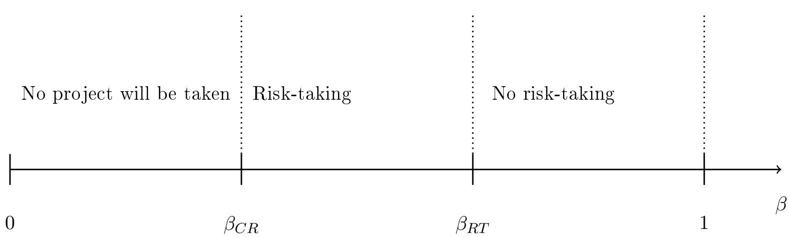

Figure 2: The bank's optimal investment choice at different collateral liquidation discount factor β

Proposition 1 shows low expected collateral liquidation discount factor could lead to excessive risk-taking or credit rationing of banks. Note that bank will play a mixed-strategy and take the risky project with probability λ when $\beta = \beta_{CR}$. As I will show in the section 3, the value of λ will be pinned down endogenously in a symmetric equilibrium which in aggregate will be the is fraction of banks being credit rationed $(1 - \lambda)$.

2.2 Endogenous liquidation value of the collateral asset

Next we study the interim problem after projects are realized. Denote $\phi \in [0,1]$ as the fraction of banks with failed projects and $(1-\phi)$ with succeeded projects. I call them 'insolvent' and 'solvent' banks respectively. Due to the perfect signal about project realization, creditors know whether their bank is solvent or not.

Solvent banks can repay their creditors by issuing some riskless securities worth r backed by the succeeded projects and thus creditors are willing to roll over their debt. Insolvent banks cannot repay their creditors at $t = 1$. Creditors face two choices: seize and liquidate the collateral asset at $t = 1$ or hold it to maturity. Obviously if creditors can all hold the asset to maturities then the illiquidity in the asset market is irrelevant. As observed in crisis period, however, creditors tend to liquidate as soon as possible or 'run for the exit' (Pedersen (2009)), leading to substantial market distress. This phenomenon can arise due to strategic considerations among creditors subject to some exogenous constraints such as maximum loss constraints (Morris and Shin (2004)) or Value-at-Risk constraints (Oehmke (forthcoming)). In practice, under SEC rules, money market mutual funds are forbidden to hold illiquid asset beyond a certain amount. Here I will abstract away from the selling decision and assume the creditors of the failed banks will choose to sell the asset at the interim date.

The market for collateral asset is illiquid at $t = 1$. To fix idea, I assume there is a representative risk-averse market maker who acts as the buyers of the collateral asset at $t = 1$. His willingness-to-pay or demand curve for the collateral is decreasing in two arguments: the measure of asset being sold and his level of risk-aversion α. Specifically, the equilibrium liquidation value function $L(\phi;\alpha)$ has the following properties[6]:

Assumption 2 *(Properties of the equilibrium collateral liquidation value function $L(\phi;\alpha)$)*

$$\frac{\partial L}{\partial \phi} < 0; \qquad \frac{\partial L}{\partial \alpha} < 0; \qquad L(0;\alpha) = v; \qquad L(\phi;0) = v; \qquad lim_{\alpha->+\infty} L(\phi,\alpha) = 0; \quad (7)$$

[6] I will offer further micro-foundation of the properties of this liquidation value function in discussion section

and $L(\phi;\alpha) \in [0, v]$*,* $L(\phi, \alpha)$ *is continuous for all* (ϕ, α)*.*

With Assumption 2, the equilibrium collateral liquidation function shows how *ex-ante banks' risk-taking decision affects ex-post fire-sale discount in the collateral market.* In any symmetric equilibrium, banks will either take the same projects $p(\beta)$ when $\beta > \beta_{CR}$ or play a mixed-strategy of choosing the risky project with probability λ when $\beta = \beta_{CR}$. Since the projects realisations are independent, the fraction of failed banks ϕ is either $(1 - p(\beta))$ or $\lambda(1 - p_2)$ respectively. Hence the higher risk the banks choose ex-ante, the steeper the fire-sale discount will be ex-post. This completes the second half of the feedback effect cycle in Figure 1.

After describing the feedback mechanism, I will characterise the equilibria and present the main results of this paper.

3 Equilibria characterisation

In any symmetric mixed-strategy equilibria, for a given equilibrium liquidation discount factor β each bank will take project $p(\beta)$ with probability $\lambda(\beta)$ according to equation (6) and

$$\lambda(\beta) = \begin{cases} 1 & \text{for} \quad \beta > \beta_{CR} \\ \lambda^* & \text{for} \quad \beta = \beta_{CR} \\ 0 & \text{for} \quad \beta < \beta_{CR} \end{cases} \tag{8}$$

where λ^* is some constant in $[0, 1]$. In aggregate there will be fraction $\lambda(\beta)$ of banks taking the project $p(\beta)$. As the projects realisations are independent, at $t = 1$ the fraction of insolvent banks and hence the measure of asset sold in the market ϕ is equal to $\lambda(\beta)(1 - p(\beta))$. Therefore, the definition of equilibrium is as follows:

Definition 1 *For any given market illiquidity parameter $\alpha \in [0, +\infty)$, a symmetric, competitive rational expectation equilibrium consists of an equilibrium liquidation discount factor $\{\beta^*\}$ such that*

1. *At $t = 0$, agents expect the liquidation discount factor to be β^* and takes it as given. A fraction of $\lambda(\beta^*)$ in (8) of banks offer the optimal short-term debt contract with yields $r(p(\beta^*), \beta^*)$ according to Proposition 1 and choose the project $p(\beta^*)$ as in (6).*

2. *At $t = 1$, fraction $\phi = \lambda(\beta^*)(1 - p(\beta^*))$ of banks become insolvent and their creditors will liquidate the asset in the market. The equilibrium liquidation value received by creditors hence is $L(\lambda(\beta^*)(1 - p(\beta^*)); \alpha)$.*

3. *In equilibrium, creditors' expectation is correct. That is, $\beta^* = L(\lambda(\beta^*)(1-p(\beta^*)); \alpha)/v$.*

I will first prove the existence of equilibrium in the next proposition

Proposition 2 *(Existence of equilibria) For any given $\alpha \in [0, +\infty)$, there always exists at least one equilibrium expected liquidation discount factor β^* satisfying the fixed-point equation:*

$$\beta^* = L(\lambda(\beta^*)(1 - p(\beta^*)); \alpha)/v \tag{9}$$

***Proof**: For a fixed α, $L(\lambda(\beta)(1 - p(\beta)); \alpha)$ is a mapping from $[0, v] \to [0, v]$. Since at $\beta = \beta_{CR}$, $\lambda(\beta) = \lambda^*$ can take any value from $[0, 1]$, $L : [0, v] \to [0, v]$ is a non-empty, convex correspondence. Thus the Kakutani's fixed-theorem applies.*

Proposition 2 suggests that depending on the market illiquidity parameter α, there could be unique or multiple equilibrium liquidation discount factor β^* which satisfy the fixed-point equation (9). Since the equilibrium liquidation discount factor β^*. The next proposition characterises under what value of α which type of equilibrium could potentially emerge. The sensitivity to small change to α in determining equilibrium as well as the multiplicity of equilibria will be interpreted as the endogenous financial fragility in this paper.

Proposition 3 *(Financial fragility and inefficiency under different degree of illiquidity in the collateral market)*

1. *for* $\alpha \in [0, \underline{\alpha}]$, *only the equilibrium with efficient project choice* (p_1) *and no credit rationing* ($\lambda(\beta_1^*(\alpha)) = 1$) *exists. The equilibrium expected liquidation discount* $\beta_1^*(\alpha) = L(1-p_1;\alpha)/v > \beta_{AS}$ *satisfies the fixed-point equation (9).*

2. *for* $\alpha \in (\underline{\alpha}, \bar{\alpha}]$, *there exists two equilibria, one with efficient project choice* (p_1) *and another with inefficient project* (p_2)*. The equilibrium expected liquidation factor in the two equilibria are* $\beta_1^*(\alpha) = L(1-p_1;\alpha)/v$ *and* $\beta_2^*(\alpha) = L(\lambda(\beta_2^*(\alpha))(1-p_2);\alpha)/v$ *which satisfy the fixed-point equation (9) and* $\beta_1^*(\alpha) \geq \beta_{AS} > \beta_2^*(\alpha) \geq \beta_{CR}$*. The equilibrium with inefficient project choice is Pareto-dominated and consists of smaller ex-ante expected payoff for banks, higher bank default risks, higher debt yields and steeper fire-sale discount of the collateral asset.*

3. *for* $\alpha \in (\bar{\alpha}, +\infty)$, *only the equilibrium with inefficient project choice* (p_2) *exists. The equilibrium liquidation value* $\beta_2^*(\alpha) = L(\lambda(\beta_2^*(\alpha))(1-p_2);\alpha)/v \in [\beta_{CR}, \beta_{AS})$ *satisfies the fixed-point equation (9).*

where $\underline{\alpha}$ *and* $\bar{\alpha}$ *are the unique values of* α *that satisfy the following equations:*

$$L(1-p_2;\underline{\alpha})/v = \beta_{AS} \tag{10}$$

$$L(1-p_1;\bar{\alpha})/v = \beta_{AS} \tag{11}$$

Credit rationing can occur in the equilibrium with inefficient project choice if the market is illiquid enough, i.e. $\alpha > \alpha_{CR}$ *where* α_{CR} *is the unique value of* α *that satisfies*

$$L(1-p_2;\alpha_{CR})/v = \beta_{CR} \tag{12}$$

For $\alpha > \alpha_{CR}$, *the equilibrium with inefficient project choice will have a equilibrium liquidation discount factor* $\beta_2^*(\alpha) = \beta_{CR}$ *and the equilibrium amount of credit rationing* $(1-\lambda^*(\alpha))$ *is increasing in* α *while* $\lambda^*(\alpha)$ *is uniquely pinned down by the equation:*

$$L(\lambda^*(\alpha)(1-p_2);\alpha)/v = \beta_{CR} \tag{13}$$

Proof: *See Appendix*

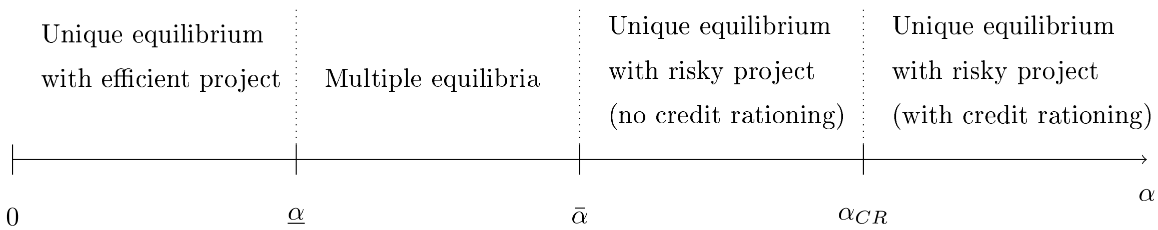

Figure 3: Equilibria characterisation under different level of illiquidity in the collateral market. Note that α_{CR} could also be in the interval $(\underline{\alpha}, \bar{\alpha}]$, depending on parameters value.

Figure 3 summarises Proposition 3 graphically. The intuition behind the unique cases are relatively straight-forward. When the market is liquid enough ($\alpha \in [0, \underline{\alpha}]$) as in case 1, creditors know that even if there are large scale liquidation ex-post ($\phi = 1 - p_2$), the expected liquidation discount factor of asset will not be too low ($\beta_2^*(\alpha) = L(1-p_2;\alpha)/v \geq \beta_{AS}$), or equivalently the requiring break-even yields of debt r will not be too high and hence banks will not have incentive to take excessive risk. Therefore, the only symmetric rational expectation equilibrium is the one with ex-ante efficient project choice (p_1). And the expected liquidation discount factor will be $\beta_1^*(\alpha) = L(1-p_1;\alpha)/v \in (\beta_{AS}, 1]$.

The case with unique inefficient risk-taking equilibrium is symmetric: where collateral asset market is so illiquid that even a small scale liquidation ($\phi = 1 - p_1$) will entail a substantial equilibrium fire-sale discount ($\beta^* < \beta_{AS}$). Thus only the contract with banks taking risky project is feasible ex-ante, leading to a unique inefficient crisis equilibrium.

The less obvious and perhaps more interesting case is when the market is moderately illiquid $\alpha \in (\underline{\alpha}, \bar{\alpha}]$ [7]. Here both equilibria could arise in a *self-fulfilling* manner from creditors' rational expectation. To see this, suppose creditors expect a low liquidation discount factor ($\beta_2^*(\alpha) < \beta_{AS}$) for the collateral asset. Creditors then will demand a high yields of debt to break-even and bank will privately choose to take the risky project (p_2). In a symmetric equilibrium this results in wide-spread bank failures ex-post ($\phi = 1 - p_2$), leading to substantial fire-sale discount ($\beta_2^*(\alpha) = L(1-p_2;\alpha)/v < \beta_{AS}$), vindicating the ex-ante

[7] consider the case with no credit-rationing, $\alpha_{CR} > \bar{\alpha}$

expectation of low liquidation discount factor. By the same mechanism, the equilibrium with efficient project choice emerges if creditors rationally expect a high liquidation discount factor.

For concreteness, I have included the plot of the fixed-point equation for the case of multiple equilibria in Figure 4.

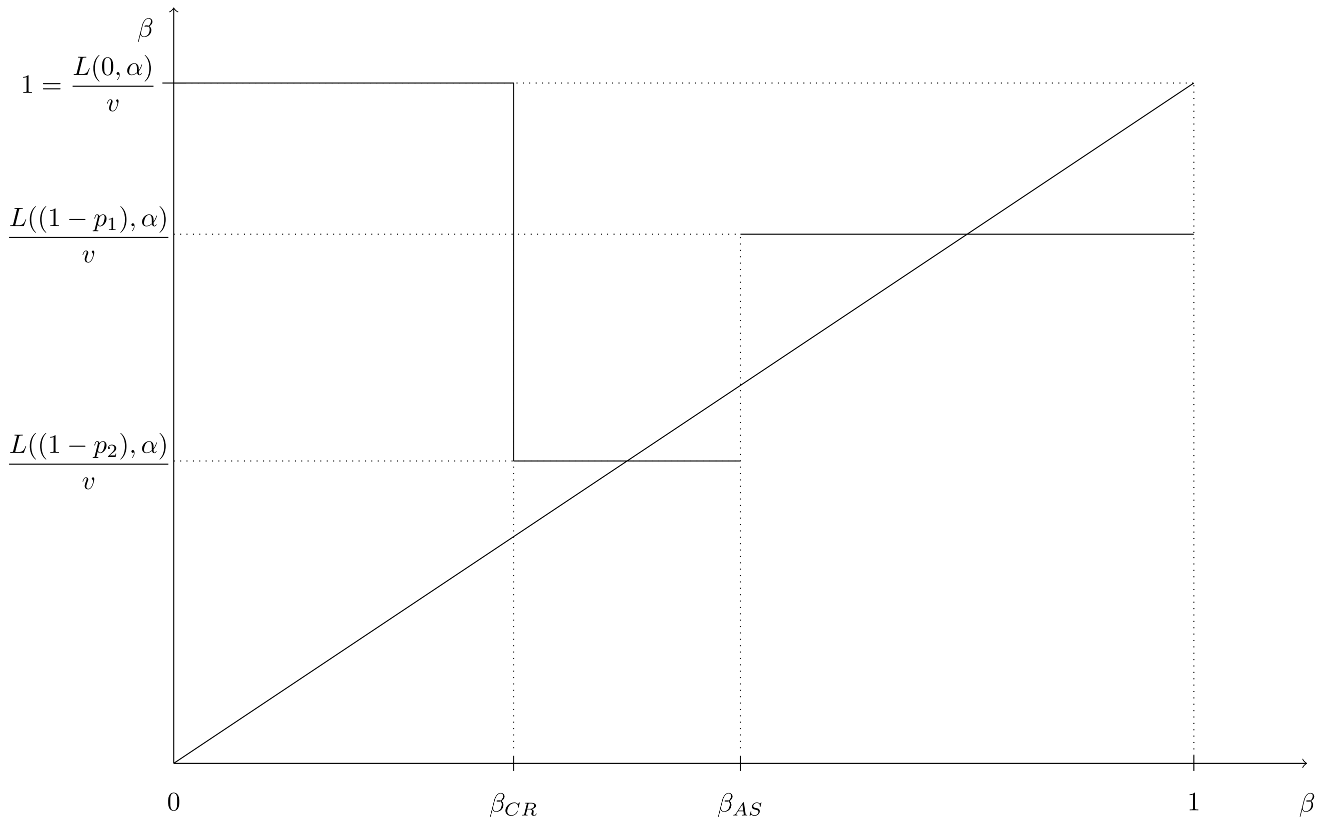

Figure 4: Plot of the fixed-point equation for the case of multiple equilibria

Credit rationing can potentially arise in the crisis equilibrium. In the case of $\alpha > \alpha_{CR}$, first note that there is no pure strategy symmetric equilibrium. Suppose in contrary that every bank decides to investment in the risky project, the equilibrium liquidation discount factor $\beta_2^*(\alpha) = L(1-p_2;\alpha)/v$ will be lower than β_{CR}, meaning that the bank would have been better off to forgo the project. On the other hand if every bank decides to forgo the project, there will not be fire-sale at the interim and thus banks would choose to invest (in the efficient project). Hence, in an equilibrium with credit rationing, the equilibrium liquidation discount factor must be β_{CR} and the bank is different between taking the risky project or forgoing it. In a symmetric mixed-strategy equilibrium for a given α, the probability of investing in the risky project $\lambda^*(\alpha)$ is the unique value that is consistent with the rational expectation, i.e. satisfying equation (13). In aggregate, $(1-\lambda^*(\alpha))$ represents the fraction of banks being credit-rationed and it is immediate that the equilibrium amount of credit-rationing is increasing in α, i.e. *the more illiquid the market is, the more credit will be rationed.*

3.1 Source of Fragility and Inefficiency

This simple model shows how credit market suddenly dries up $(\lambda(\beta^*))$ and how discontinuous jump in banks default risks $(1-p(\beta^*))$, short-term debt yields $r(p(\beta^*), \beta^*)$, and fire-sale discount of the collateral asset $(1-\beta^*)$ can occur, due to the discrete change in expectation of equilibrium liquidation discount factor β^*. These consequences of switching from the efficient equilibrium to the inefficient risk-taking equilibrium are broadly consistent with the stylised facts of the Global Financial Crisis 2007-2009 and the recent European Sovereign Debt turmoil. These drastic changes can manifest in two ways: when the market is moderately illiquid $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, equilibrium can switch from the relatively more efficient one to the crisis equilibrium due to the 'rational pessimism' of the market. Alternatively, suppose $\alpha = \bar{\alpha}$ and the existing equilibrium is the efficient one, a small increase in market illiquidity will switch the equilibrium to the unique crisis equilibrium. Both kinds of switch will result in large jumps in the financial markets.

The multiplicity of equilibria result is reminiscent to the classic coordination failure bank run problem in Diamond and Dybvig (1983). Nonetheless, the mechanism here is different. In Diamond and Dybvig (1983), depositors within a bank fails to coordinate on the action to whether withdraw the deposit or not and the withdrawal exerts an externality on other depositors due to costly liquidation. Here the coordination failure occurs at the belief of creditors in *different banks* about the size of liquidation discount of the collateral and the pecuniary externality from one bank to another acts through the illiquidity of the collateral market.

This result also represents fragility in the banking system and potential welfare loss. Note that the equilibrium with efficient project choice Pareto-dominates the one with risky project. While creditors always break-even, banks get a smaller surplus in the equilibrium with risky project choice. The inefficiency comes from two sources: the smaller NPV in the risky project and the transfer of risky asset from risk-neutral banks (and creditors) to the risk-averse market maker.

The next section contains a numerical exercise to illustrate the main results more concretely.

3.2 Numerical Example

In this numerical exercise, the values of the set of parameters $\{v, p_1, p_2, X_{p_1}, X_{p_2}\}$ are chosen to be $\{0.8, .0.9, 0.8, 1.163, 1.28\}$ to satisfy all the assumptions and parameter restrictions mentioned in the paper. With this set of parameters value and the liquidation value function $L(\phi; \alpha) = e^{-\alpha\phi}v$, the threshold values $\{r_{AS}, \beta_{AS}, \beta_{CR}, \underline{\alpha}, \bar{\alpha}, \alpha_{CR}\}$ are $\{1.027, 0.946, 0.85, 0.276, 0.552, 0.812\}$. As my main interests are in fire-sale discount of the collateral value, this specification implies a 5.4% and 15% expected fire-sale discount of the collateral value will cause asset-substitution and credit-rationing respectively[8]. I will discuss the multiple equilibria case and the case of crisis equilibrium with credit rationing in details.

Multiple equilibria $\alpha \in (\underline{\alpha}, \bar{\alpha}]$: Set the market illiquidity parameter $\alpha = 0.4$. As expected, there are multiple equilibria. The results are listed in the Table 1.

Comparing to the efficient equilibrium, the crisis equilibrium consists of higher bank default risk, higher debt yields, and larger collateral fire-sale discount. In particular the equilibrium collateral fire-sale discounts are just below and above the asset-substitution threshold 5.4% respectively and hence both equilibria are consistent with rational expectation. In terms of welfare, as creditor always break-even in the model, the bank net profit

[8] $1 - \beta_{AS} = 5.4\%$ and $1 - \beta_{CR} = 15\%$

Table 1: Numerical example with multiple equilibria ($\alpha = 0.4$)

Equilibrium Values	Efficient Equilibrium	Crisis Equilibrium
Bank default risk $(1 - p(\beta^*))$	10%	20%
Collateral fire-sale discount $(1 - \beta^*)$	3.92%	7.69%
Short-term debt net yields $(r(p(\beta^*), \beta^*) - 1)$	2.57%	6.54%
Bank (per dollar) net profit from investment	4.36%	1.17 %

from investment, i.e. NPV of investment net of expected fire-sale cost, represents the social welfare surplus and again the crisis equilibrium entails some welfare loss.

Crisis equilibrium with credit rationing $\alpha > \alpha_{CR}$**:** Set $\alpha = 1$. The equilibrium will consist of mixed-strategy and the fraction of banks which undertake the risky project is $\lambda^* = 81.26\%$. In other words, there are about 19% of the banks being credit-rationed. It can be easily checked that the equilibrium collateral liquidation discount factor $L(\lambda^*(1 - p_2); \alpha)/v$ equals to the credit-rationing threshold β_{CR} and hence the bank net profit from investment equals to zero.

3.3 Discussion

Before I move on to the policy implication, let me discuss some key elements of the model.

Illiquidity in collateral market Although distressed asset sales are quite common in the literature and the real world, its importance in the paper warrants some discussion about its context and justification. There are at least two ways to justify this downward-sloping demand schedule:

- *A representative risk-averse market maker*

  One can regard the potential buyers of the collateral at the interim as a risk-averse market maker. When the market maker buys a large amount of the collateral asset with a common risk factor, he has to raise a considerable amount of fund in short notice to execute the sell orders he receives and hedge his long position. This fresh funding can be costly and the market maker will be *de facto* risk-averse if he is subject to some financing constraint such as value-at-risk imposed by his financiers. A competitive market maker who needs to break-even thus has a downward sloping willingness-to-pay for these collateral asset. A similar assumption can be found in Morris and Shin (2004); Bernardo and Welch (2004).

- *Required returns for long-term investors with scarce capital*
  Alternatively, the potential buyers of the collateral can be some patient, long-term investors who are willing to buy and hold these assets to maturity. Typical example of these investors are pension funds or hedge funds. However they have limited amount of capital and thus holding cash at $t = 0$, rather than investing in some other productive projects, entails opportunity cost. If the marginal cost of holding cash or equivalently the marginal return of the forgone projects are increasing, more supply of the asset requires a deeper fire-sale discount and hence expected return for the long-term investors to hold more cash. Similar setup can be found in Diamond and Rajan (2011); Stein (2012).

Collateralized short-term debt The assumption that banks rely on collateralised short-term debt is a close description of modern financial system. The above setup aims to capture the fact that in practice the majority of financial intermediation between financial institutions and institutional lenders nowadays are collateralised and short-term. One of the reasons for its popularity is that secured and short-term lenders are senior to other creditors and in the time of the borrower's bankruptcy, they can own and access the asset without going through the time-consuming and costly bankruptcy procedure[9]. It also allows creditors to adjust their terms of lending or withdraw completely according to the information revealed about the borrowers' financial health (see, e.g. Brunnermeier and Oehmke (2012)). Finally using secured borrowing such as repo and ABCP allow financial institutions to tap into the relatively cheap funding from cash-rich short-term creditors like

[9]For example, the settlement of Lehman Brothers' creditors took about 3 years and the recovery rate of the debt issued by Lehman Brothers is about 8 cents on a dollar. (Morris and Shin (2009) page 8).

Money Market Funds (MMF) (Shin (2009)).

Banks and hedge funds which borrow in repo transaction and ABCP conduits are essentially using their assets as collateral to obtain finance with a short maturity. Money Market Funds (MMF) is a typical cash-rich party providing credits in such deals. Type of collateral used includes Treasury, Agency and non-Agency ABS/MBS, corporate bonds, etc. See Krishnamurthy, Nagel, and Orlov (2012) for a detailed empirical report.

Maturity mismatch Maturity mismatch between the asset (project and collateral) and the liability (short-term debt) is the essential source of friction in this paper. Should either the cashflow of the project or the dividend of the collateral mature at $t = 1$, the illiquidity of the collateral market will not arise.

Market illiquidity In the model if the creditors can hold the collateral asset till $t = 2$ or the asset market is perfectly liquid, the inefficient risk-taking equilibrium will not arise. Interim liquidation can be motivated by avoidance of mark-to-market loss as in the model, or the fact that according to SEC rules MMF can only hold a certain amount of illiquid or long term securities. Asset market illiquidity can be motivated by the limit of arbitrage hypothesis, which suggest the marginal buyer of the asset is a specialized investor who is either risk-averse or financially constrained. Gabaix, Krishnamurthy, and Vigneron (2007) provide theory and evidence in support of the limit of arbitrages hypothesis in MBS market. Under this interpretation, the market illiquidity arises from the funding illiquidity of specialized arbitrageurs.

Early trading of collateral asset at $t = 0$ The benchmark model could alternatively allow the possibility of early sale of collateral asset at $t = 0$. As noted by Diamond and Rajan (2009), however, the banks will not want to sell their potentially illiquid asset ex-ante, i.e. "a seller strike". The reason is that the buyers who anticipate a fire-sale at interim are only willing to buy the collateral at fire-sale price, even at $t = 0$. Meanwhile the seller will be strictly better off to hold on to the asset as the project may succeed and hence no fire-sale will occur eventually.

4 Implication for Central Banks intervention

The aforementioned inefficiency and the unpredictability of financial crisis create a potential role for government to intervene. This section discusses how central banks can act as a social planner to reduce the potential welfare loss associated with excessive risk-taking and massive liquidation of assets at fire-sale price. In particular I will discuss a few unconventional intervention policies employed by central banks especially the Federal Reserve during the Global Financial Crisis 2007-2009.

4.1 Market Maker of Last Resort

In light of the turmoil in the Asset-Backed Securities and Commercial Paper market in Autumn 2007, Buiter and Sibert (2007) have suggested that central banks around the world should act as 'Market Makers of Last Resort', taking unconventional if necessary measures to guarantee the proper functioning of some key important asset markets. Here I will discuss two policy under this role: Asset Price Guarantee and Collateral Swap.

Asset Price Guarantee The main fragility and inefficiency in the model is the multiple equilibria case where creditors' rational fear of the fire-sale makes it happen. Central bank can eliminate this belief by ex-ante committing to buy any amount of asset at a price $\beta^{CB}v$ higher than $\beta_{AS}v$. Knowing one can always go to the central bank as the Market Maker of Last Resort to liquidate asset, any expected liquidation factor below β^{CB} cannot be sustained in a rational expectation equilibrium. Hence only the efficient equilibrium can emerge.

If the asset price guarantee $\beta^{CB}v$ is chosen judiciously, i.e. $\beta^{CB} \in (\beta_{AS}, \beta_1^*(\alpha))$, the price support facility, and thus public funding, will not be used in equilibrium. This is because in the efficient equilibrium, liquidating in the market gives expected liquidation value $\beta_1^*(\alpha)v$ higher than the price offered by the central bank. Also notice that the central bank's committed price $\beta^{CB}v$ is less than the fair value v and thus the purchase commitment is credible as long as the social cost of using public funding at $t = 1$ is not too high.

One can argue that the Outright Purchase Transaction launched by the European Central Bank in September 2012 has similar features of the asset price guarantee policy outlined above. Subject to certain conditions, the OMT allows the ECB to direct participate in the secondary sovereign bond market of their member countries to "safeguard an appropriate monetary policy transmission". One notable feature is that there is "no ex ante quantitative limits are set on the size of Outright Monetary Transactions."[10]

Collateral Swap The negative feedback loop in this model will not arise if the collateral is perfectly liquid. Hence if central banks can swap the potentially illiquid collateral with some liquid collateral such as Treasuries, the fire-sale equilibrium will not arise. Indeed in March 2008 the Federal Reserve implemented the Term Securities Lending Facility (TSLF) which allowed primary dealers to borrow Treasuries against some eligible collateral such as AAA Residential Mortgage-Backed Securities. The borrowing rate was determined by auctions. Of course if the Treasuries are deemed riskless, the degree of illiquidity will be close to zero and no fire-sale will happen. The cost involved for this intervention will be the issuance and fiscal cost of these government securities.

4.2 Lender of Last Resort

Direct lending Another approach central bank could do is to provide liquidity directly. For instance, central bank could directly be the financiers of the banks, circumventing the money market mutual funds. The main advantage of the government is that without liquidity or regulation concern, it can hold the asset to maturity, avoiding any fire-sale cost at the interim. In the Federal Reserve's Term Asset-Backed Securities Loan Facility (TALF), borrower with eligible collateral assets can borrow three to five years term loans. However, in this case public funding will be involved and the size and the riskiness of balance sheet of the central banks will increase.

[10]Source: ECB Press Release - "Technical features of Outright Monetary Transactions", September 6th, 2012

5 Concluding remarks

This paper shows a novel form of financial fragility stemming from the feedback effect between the risk-taking incentives of financial agents (e.g. banks) and the illiquidity in the collateral asset market. When banks collateralise their asset to borrow in the form of short-term debt such as repo and asset-backed commercial paper, there is a feedback loop between banks' ex-ante risk-taking decisions and the ex-post fire-sale discount of collateral asset market. Fragility arises in the form of self-fulfilling, multiple equilibria whereas the inefficient equilibrium consists of high endogenously chosen risks, wide-spread insolvency, and steep fire-sale discount of the collateral.

This paper provides economic rationale for central banks to intervene in the collateral market. When the market is moderately illiquid, asset price guarantee can costlessly eliminate rational fear of fire-sale of the market participants and hence inefficient crisis equilibrium could not arise. When the market is stuck at the unique inefficient equilibrium, central banks can provide liquidity in the market by direct asset purchase, collateral swap and direct lending can restore the efficient equilibrium at a fiscal cost.

Appendix: Proof

Proof of Lemma 1:

First observe that (PC) will be binding at optimal, otherwise bank can decrease r by a small amount to increase profit. Substituting (RE) into (PC), one will obtain the expression of $r(p_i, \beta)$. Notice that $r(p_i, \beta)$ is decreasing in both β and p_i. Plugging in $r(p_i, \beta)$ in the (IC) it will become a fixed point equation. For $\beta < \beta_{AS}$, p_2 is the unique fixed point to satisfy (IC), thus $r(p_2, \beta)$ is the only feasible choice and hence the solution of the maximization problem. For $\beta \geq \beta_{AS}$ there are two cases: For β large enough p_1 is the unique fixed point while for intermediate β, both p_1 and p_2 satisfy the (IC) fixed point equation. In either case, $r(p_1, \beta)$ maximizes bank's expected payoff. QED

Proof of Proposition 3:

I will first prove the proposition under the case that $\alpha_{CR} > \bar{\alpha}$, hence without considering the possibility of credit rationing. Then I will characterise the case with credit rationing.
First note $L(\phi; \alpha)$ being continuous and strictly decreasing in $\phi \in (0, 1)$ and $\alpha \in (0, +\infty)$, $(1-p_1) < (1-p_2)$ implies $\underline{\alpha} < \bar{\alpha}$. Denote the two potential equilibrium expected liquidation discount factor $L(1-p_1; \alpha)/v$ and $L(1-p_2; \alpha)/v$ as $\beta_1^*(\alpha)$ and $\beta_2^*(\alpha)$ respectively.
To see case 1, by the definition of $\underline{\alpha}$, for $\alpha \in [0, \underline{\alpha}]$, even when the measure of liquidated asset is anticipated to be $(1-p_2)$, it is not incentive compatible for bank to take the risky project as $L(1-p_2; \alpha)/v \geq \beta_{AS}$;. Hence for $\alpha \in [0, \underline{\alpha}]$, only the equilibrium with safe project can exist. Similar argument works for case 3 to show that $\beta_1^*(\alpha)$ cannot be a solution of the fixed-point equation by using the definition of $\bar{\alpha}$.
For case 2 for $\alpha \in (\underline{\alpha}, \bar{\alpha}]$, $\beta_1^*(\alpha) \equiv L(1-p_1; \alpha)/v \geq \beta_{AS} > L(1-p_2; \alpha)/v \equiv \beta_2^*(\alpha)$. It is immediate to check that both $\beta_1^*(\alpha)$ and $\beta_2^*(\alpha)$ satisfy the fixed-point equation (9), using the project choice function $p(\beta)$ in (6). Thus there are multiple equilibria.
Finally the case with credit rationing. For $\alpha > \alpha_{CR}$, $L(1-p_2, \alpha)/v < \beta_{CR}$. Thus by the continuity and strict decreasing property of $L(., \alpha)$ in ϕ, there exist a $\lambda(\alpha) \in [0, 1]$ that satisfies the equation (13). As L is decreasing in α, by total differentiating equation (13) one can see $\lambda(\alpha)$ is decreasing in α.

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