Bailout Uncertainty in a Microfounded General Equilibrium

Model of the Financial System

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Abstract

This paper develops a micro-founded general equilibrium model of the financial system composed of ultimate borrowers, ultimate lenders and financial intermediaries. The model is used to investigate the impact of uncertainty about the likelihood of governmental bailouts on leverage, interest rates, the volume of defaults and the real economy. The distinction between risk and uncertainty is implemented by applying the maxmin with multiple priors framework to lenders' beliefs about the probability of bailout.

Results of the analysis include: (i) An unanticipated increase in bailout uncertainty raises interest rates, the volume of defaults in both the real and financial sectors and may lead to a total drying up of credit markets. (ii) Lower exante bailout uncertainty is conducive to higher leverage - which raises moral hazard and makes the economy more vulnerable to expost increases in bailout uncertainty. (iii) Bailout uncertainty raises the likelihood of bubbles, the amplitude of booms and busts as well as the banking and the credit spreads. (iv) Bailout uncertainty is associated with higher returns' variability in diversified portfolios and higher systemic risks, (v) Expansionary monetary policy reinforces those effects by inducing higher aggregate leverage levels.

Keywords and Phrases: Risk, Uncertainty, Lehman's default, Leverage, Financial intermediaries, Bailouts, Duration mismatches.

JEL Classification Codes: G01, G11, G2, G18, E3, E4, E5, E6, D81, D83.

1 Introduction

Financial sector bailouts in the US and more recently in Europe have revived the well known dilemma between restoration of confidence in the face of a panic and the costs of moral hazard. On one hand, when a panic engulfs financial markets, bailouts appear indispensable in order to restore confidence and prevent further collapses in the financial system. On the other, by subsidizing opportunistic behavior at the expense of taxpayers, bailouts encourage excessive risk taking on the part of financial institutions, borrowers and lenders, and plant the seeds of the next bubble.

Different experts in both policymaking circles as well as in academia often find themselves at odds regarding the ways to handle this problem. In spite of currently ongoing reforms in regulation this dilemma is, therefore, likely to be a central issue during the upcoming decade. Whether, and how exactly will bailout policies be deployed in the future is largely an open issue. However, due to the lack of consensus about the precise ways to deal with the (exante and expost) trade-offs induced by bailouts, it is extremely likely that bailout uncertainty is likely to be non negligible in the foreseeable future. The 2008 bailout zigzags in the US (Bear-Stern versus Lehman) and current uncertainties about the reaction of EMU governments to sovereign debt problems in Greece, Cyprus and Spain attest to that.

This paper develops a micro-founded general equilibrium model of the financial system and uses it in order to investigate the impact of an increase in bailout uncertainty on financial markets and the real economy. It also investigates the exante, leverage expanding, moral hazard problems created by perceived generous governmental bailout policies.

As is well known since Knight's (1921) work risk and uncertainty are distinct concepts. Modern formulations of this distinction in the context of pecuniary returns conceptualize risk as some measure of spread for a **known distribution** of the stochastic return. Uncertainty, on the other hand, is a situation in which individuals are **unsure about the probability distribution** of returns and entertain the possibility that several alternative probability distributions have positive measure. An increase in uncertainty is then viewed as a an enlargement of the set of plausible probability distributions with positive measure. Ellsberg (1961) and others have demonstrated by means of

experiments that individuals are averse to ambiguity in the sense that, other things the same, they prefer a lottery with a known probability distribution to a lottery in which several distributions are believed to be possible.

Gilboa and Schmeidler (1989) (GS in the sequel) conceptualize an investor's uncertainty by postulating that he possesses a subjective set of probability measures, or multiple priors, over outcomes.[1] Their framework implies that for each possible action the investor assumes that the worst (by the expected utility criterion) possible distribution will realize and chooses his action so as to attain maximum expected utility over this set of worst outcomes. This paper utilizes the GS notion of uncertainty and the associated max-min behavioral criterion to analyze the impact of an increase in uncertainty about governmental bailout policy on financial markets, the aggregate level of credit and, through them, on the real economy.

Prior to Lehman's collapse financial market's beliefs about the probability of bailout have been relatively optimistic due to Bear-Stern's bailout in March 2008 as well as to the implicit US government guarantees of Fannie Mae and Freddie Mac's liabilities (Meltzer (2009)). In terms of the GS framework this means that the family of binomial bailout distributions with positive mass was concentrated in the relatively high range of bailout probabilities. After Lehman's collapse in mid September 2008 this range expanded downward toward bailout probabilities that previously were given zero mass.[2]

The behavior of credit default swap (CDS) spreads during the two weeks following Lehman's collapse provides a dramatic illustration of the sensitivity of bailout perceptions to public signals. In the aftermath of this collapse credit markets experienced substantial waves of deleveraging, totally drying up in some cases, and both the level and variability of CDS spreads went through the roof. Table 1[3] shows the behavior of Citibank's CDS spread index during the period just preceding

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[1]The recent literature on robustness is also based on the notion of multiple priors (Hansen and Sargent (2008))

[2]Note that although such expansion of beliefs involves assignment of positive mass to bailout probabilities that had zero mass prior to Lehman's collapse the latter is not quite a "black swan" event (Taleb (2007)). Taleb's black Swan is an event whose perceived probability had zero mass (or this event has not even been considered in the state space due to unawareness) prior to its initial realization. By contrast the perceived probability of no bailout prior to Lehman's collapse was not zero. However, after this event some bailout distributions that previously were assigned zero mass entered the set of bailout distributions with positive mass.

[3]Source: Cochrane and Zingales (2009) .

Lehman's default and the final approval of the TARP bailout package at the beginning of October 2008.

Date	Event	CDS Spread
13-14/9		150
15/9	Lehman files for chapter 11	
16-17/9	Paulson suggests TARP to Congress	250
18-19/9		150
22-23/9	Paulson & Bernanke address Congress	450
24-25/9		350
29/9	Congress rejects TARP proposal	Almost 450
3/10	Amended TARP approved by Congress	
5-10/10	Aftermath of approval	150

Table 1: **Chronology of CDS spread around Lehman's collapse**

The table demonstrates the high sensitivity of the CDS spread to ongoing public signals about the likelihood of bailouts. In particular the spread strongly reacts to events like the rejection of the proposed TARP bailout package by Congress in September 2008 and its final approval in early October. This supports the view that financial markets participants are quite sensitive to news about the likelihood of bailouts.

Following Lehman's collapse and the ensuing public debate among policymakers about the wisdom of governmental bailouts two things happened. Public uncertainty (also known as ambiguity in the decision theory literature) about the likelihood of bailouts increased and so did the public's aversion to this uncertainty (or ambiguity aversion in the terminology of decision theory). Importantly, beliefs are not the only determinant of an individual's subjective set of priors, but also her preferences concerning ambiguity. That is, this set need not be the set of priors that is literally deemed possible by the individual. In particular, suppose two individuals share the same subjective information, i.e., they both believe the same set of bailout probabilities are possible. Then modern decision theory implies that the set of multiple priors of the **less** ambiguity averse individual is a **subset** of the set of multiple priors of the **more** ambiguity averse individual (See, for example Ghirardato and Marinacci (2002, Theorem 17-(ii)) and Klibanoff, Marinacci, and Mukerji (2005, p. 1872)).

Thus, expansion of the set of multiple bailout priors in the aftermath of Lehman's collapse may reflect more ambiguous beliefs or a higher degree of ambiguity aversion or both. Obviously both factors reinforce each other. Our sense is that, in the aftermath of the financial trauma caused by Lehman's default the increase in ambiguity aversion might have been more important and more persistent for reasons elaborated two paragraphs below. Be that as it may, the upshot is that following this episode the set of multiple priors about the probability of bailouts expanded. As a consequence the lower bound on the set of binomial distributions with perceived positive mass went down. The paper shows that lenders' expected utility is lower the lower is the probability of bailout. In conjunction with the GS max-min criterion the expansion in the set of multiple priors implies that, once atraumatic event like Lehman's collapse materializes, lenders become more reluctant to lend either because the ambiguity of bailouts has increased, or because ambiguity aversion has increased, or both.

Since, by the max-min behavioral criterion individuals ultimately act in each period on the basis of only one bailout distribution their micro optimization problems collapse to maximization of expected utility given a **unique** probability of bailout in each period. Consequently, results identical to ours also obtain in the presence of only bailout risk provided the unique bailout probability is taken to be the worst probability distribution both prior to and following Lehman's collapse. This begs the question of why we choose to utilize the more cumbersome multiple priors framework rather than the simpler and more standard "risk only" framework.

The answer has two parts. First, following Keynes, we believe that the impacts on financial markets of major, largely unanticipated, events like the Lehman episode and its immediate aftermath are not adequately captured by the risk only framework.[4] Since there is no single objective probability that government will repay the debts of failing financial institutions it is natural for individuals to

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[4]Keynes (1937, pp. 114-115)writes "*Now a practical theory of the future [...] has certain marked characteristics. In particular, being based on so flimsy a foundation, it is subject to sudden and violent changes. The practice of calmness and immobility, of certainty and security, suddenly breaks down. New fears and hopes will, without warning, take charge of human conduct. The forces of disillusion may suddenly impose a new conventional basis of valuation. All these pretty, polite techniques, made for a well-panelled board room and a nicely regulated market, are liable to collapse. At all times vague panic fears and equally vague and unreasoned hopes are not really lulled, and lie but a little way below the surface.*"

believe that several bailout probability distributions are possible and to adjust the set of such priors in line with actual developments and news about government's intentions. The modern multiple priors framework provides an operational way to capture this uncertainty and to reduce its complexity to manageable proportions. Second, since, as reported above an increase in ambiguity aversion induces an enlargement of the set of priors, such an enlargement and the associated lower minimal probability of bailout also capture increases in ambiguity aversion following salient traumatic events.

Two recent real life episodes are consistent with this view. First, according to the Non-Life Insurance Rating Organization of Japan (NLIRO) stricken Fukushima prefecture, bearing the brunt of the huge earthquake and tsunami that devastated the region at the beginning of 2011, saw rates of new earthquake insurance coverage increase almost threefold in the aftermath of the earthquake (Majirox news, August 24 2011). Provided this event did not appreciably change the beliefs of individuals about the objective probability distribution of such events this evidence is consistent with the view that, following the trauma caused by the tsunami, the set of multiple priors concerning such events expanded mainly because of an increase in ambiguity aversion rather than in ambiguity.

Second, the risk only approach appears to be inconsistent with the evolution of events over the entire four years since Lehman's collapse. Although it is reasonable to expect that during the first several months following this event the (assumed single) bailout probability was adjusted downward it is harder to maintain such a position later on. The reason is that since Lehman's collapse US policymakers introduced various measures, such as TARP and numerous quantitative easing programs, that most likely, have led the public to believe that the probability of bailout moved back up. In spite of this, the share of total US banks reserves in total banking credit that increased by a factor of about 20 upon Lehman's collapse (from half a percent to 12 percent), remained in the range of this elevated level at least till the beginning of 2012 (Cukierman (2013)). This behavior is inconsistent with a story of a single bailout prior that first went down and subsequently back up. By contrast it is consistent with a persistent change in ambiguity aversion within a multiple priors framework.

We turn next to a brief overview of the main structure of the paper. The financial system is

composed of large numbers of each of the following: Ultimate borrowers (investors), fully diversified ultimate lenders (like pension and mutual funds) and financial intermediaries that borrow from the lenders and lend to investors. Leverage levels assumed by borrowers and extended to them by financial intermediaries depend on the cost of credit and on (rational) expectations about the future cost of credit. Since returns to investors are stochastic some of them default and do not repay their debts to financial intermediaries. As a consequence some of the intermediaries are unable to repay their debts to lenders. Government bailout policy consist in (stochastically) repaying the debt of failing intermediaries to lenders. Beliefs about the likelihood of such bailouts affect the amount of credit offered exante to financial intermediaries by lenders, and through them on the amount of credit offered by the latter to borrowers and on the general equilibrium level of interest rates. Thus, in spite of the fact that potential bailouts only target loans from lenders to financial intermediaries, government's stochastic bailout policy affects leverage and interest rate throughout the entire financial system.

The main objectives of the paper are: *(i)* To identify the mechanisms through which beliefs about bailout policy affect short term credit within the financial system, interest rates and credit to the real sector. *(ii)* To trace out the impact of an expost (after long term investment decisions have been made) change in bailout perceptions on interest rates and the volume of defaults throughout the entire financial system. *(iii)* To analyze the impact of expansionary monetary policy on leverage and risk appetite. The paper's framework makes it possible to trace out both the exante and the expost consequences of beliefs about the generosity of bailout policies. Exante, a more generous bailout policy increases moral hazard in all segments of the financial system and induces an overall expansion of credit.[5] But expost the maintenance of a generous bailout policy may become necessary just to avoid a crisis even if government no longer desires to maintain high bailout levels.

Important features of the model include: *(i)* An individual tradeoff between return seeking through higher levels of leverage and higher probability of total loss at the **individual level**. *(ii)* Exante and expost relations between the worst probability of bailout and leverage at the **aggregate**

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[5]Borio C. and M. Drehmann (2009) convincingly argue that such a credit buildup raises the likelihood of a financial crisis.

level. *(iii)* **Duration mismatches:** Borrowers need financing for two periods but get only one period loans from financial intermediaries in each period. *(iv)* The model's focus is on the segment of **the shadow banking system** (like SIV and hedge funds) in which funds are secured only for short periods. Accordingly, financial intermediaries are assumed to borrow for only one period.

The rest of the paper is organized as follows. Section 2 presents a general overview of the model. Sections 3, 4 and 5 introduce a typical borrower, a typical financial intermediary and a typical lender and characterize the optimal microeconomic behavior of each type of agent. Government's bailout policy is specified in Section 5. General equilibrium of the financial system and the determination of market rates are discussed in section 6. Section 7 analyzes the impact of an exogenous decrease in perceptions about the likelihood of bailout on financial markets. Section 8 discusses the exante choice of leverage by borrowers in general equilibrium including, in particular, the impact of perceived bailout policy and the associated moral hazard problem. Section 9 reflects on the social desirability of exante commitments to a particular bailout policy. This is followed by concluding remarks in Section 10. A central result of the paper (implied by the discussion in sections 7 and 8 and elaborated in the conclusion) is that higher bailout uncertainty raises the amplitude of booms and busts. Most proofs are in the Appendix.

2   Framework

There is a large number of each of the following risk averse (identical within each group) 3 types of agents: Borrowers (B), Financial intermediaries (F) and Lenders (L) each possessing one unit of equity capital.[6] The initial masses of each type of agent are M_B, M_L and M_F for borrowers, financial intermediaries and lenders, respectively.[7]

There are 3 time periods labeled 0, 1 and 2. Only borrowers-investors have access to real investment decisions. All such decisions are made by them in period 0 and are long term in the sense, that

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[6]We use the following notational conventions: the subscript $j = \{B, F, L\}$ to a variable x_{jt} indicates the agent type, and subscript $t = \{0, 1, 2\}$ indicates time. When the time index is omitted the variable refers to any of the time periods between 0 and 2. Random variables are identified by a tilde on top of the variable (e.g. $\widetilde{X}$).

[7]The financial markets model in the paper can be thought of as a microfounded version of general equilibrium approaches to monetary theory and policy (Brunner and Meltzer (1997) , Tobin (1969)).

once chosen, the project's size cannot be adjusted. The selected project size is $(1 + L_B)$, where 1 is the borrower's initial equity capital and L_B is the leverage he selects to take. Only short term loans are available to borrowers with interest rates r_{B1} and r_{B2} for loans assumed in the first and in the second period, respectively. Interest rates on loans and project's yields are all specified in terms of net returns.

Each borrower can get loans only from financial intermediaries. The amount of leverage, L_B, demanded by a borrower is determined as a function of r_{B1} and of expected r_{B2}, by means of individual optimization. Each financial intermediary can obtain short-term funds, L_F, from lenders. The intermediaries generally splits his total funds $(1 + L_F)$ between a fraction z_F allocated to a partially diversified portfolio of loans to borrowers and a fraction $(1 - z_F)$ allocated to a risk free asset that pays a fixed interest rate r_f.[8] The return on the risk free asset, r_f, is determined by the monetary authority.

Financial intermediaries pay to lenders short term interest rates r_{L1} and r_{L2} in periods 1 and 2 (for loans taken in periods 0 and 1) respectively. A typical lender splits his initial wealth of 1 between a fraction z_L of funds allocated to loans to financial intermediaries and a remaining fraction, $(1 - z_L)$, that is invested in the risk free asset. In contrast to a typical financial intermediary, whose portfolio of loans to borrowers is only partially diversified, a typical lender holds his selected portion of loans to financial intermediaries in a fully diversified portfolio of loans.

The supply of loans to borrowers by an individual financial intermediary and his demand for loans from lenders, L_F, are determined through the intermediary's individual optimization as a function of the interest rates r_{B1}, r_{B2}, r_{L1} and r_{L2}. Those interest rates are determined through general equilibrium competitive clearing in periods 0 and 1 respectively in two markets within each period: The market for loans from intermediaries to borrowers and the market for loans from lenders to financial intermediaries.[9]

As elaborated in the next section, returns on real projects are stochastic and therefore risky.

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[8]An intermediary's portfolio is partially diversified in the sense that only part of the idiosyncratic risk is eliminated. By contrast in a fully diversified portfolio all idiosyncratic risk is eliminated and only the systematic risk remains.

[9]A loan carrying interest rate r_t $(t = 1, 2)$ is contracted in period $t - 1$ and settled in period t.

They realize in period 2. A real-project yield, $\widetilde{Y}_P$, depends on two independent random variables: an aggregate economy-wide shock, $\widetilde{Y}_A$, and a specific (idiosyncratic) shock, $\widetilde{Y}_I$. The realization of the aggregate shock is not revealed prior to period 2. Although all idiosyncratic shocks realize only in period 2, the value of this future realization becomes publicly known for some borrowers already in period 1. Depending on the information available in period 1 borrowers can be classified to one of the following three groups: Lucky borrowers for whom it becomes known they will get a high $\widetilde{Y}_I$, unlucky borrowers for whom it becomes known they will get a low $\widetilde{Y}_I$, and regular borrowers for whom no advance return information is available in period 1. The availability of such information is important because it affects the borrower's ability to get refinancing in period 1. Since project yield is random and borrowers have some leverage obligations they generally may default in either of periods 1 or 2. A borrower defaults in period 1 if he does not succeed in securing credit to carry over his project on to period 2. He defaults in period 2 if the total final project return does no suffice to service the debt incurred in the previous period.

A financial intermediary can also default in period 1 or 2 if the principal and the interest rate paid to him by borrowers cannot cover his obligations to lenders. When a financial intermediary defaults lenders lose their entire investment in this intermediary including the principal and the interest rate. Government can possibly and selectively pay the debt of defaulting financial intermediaries to lenders. But governmental bailout policy is uncertain in the Knigthian sense. More precisely, individuals entertain multiple priors about the probability of bailout, or in the language of modern decision theory – government's bailout policy is ambiguous.

Figure 1 presents a bird's eye view of the model's financial system. In the figure z_F and z_L represent the fractions of funds Fs and Ls allocate to risky loans, and r_B and r_L are the rates paid by Bs and received by Ls respectively. $\widetilde{Y}_A$ and $\widetilde{Y}_I$ are aggregate and individual components of the total net return to a typical borrower.

3   The Typical Investor-Borrower (B)

This section presents the borrower's problem. First it specifies B's real investments opportunities
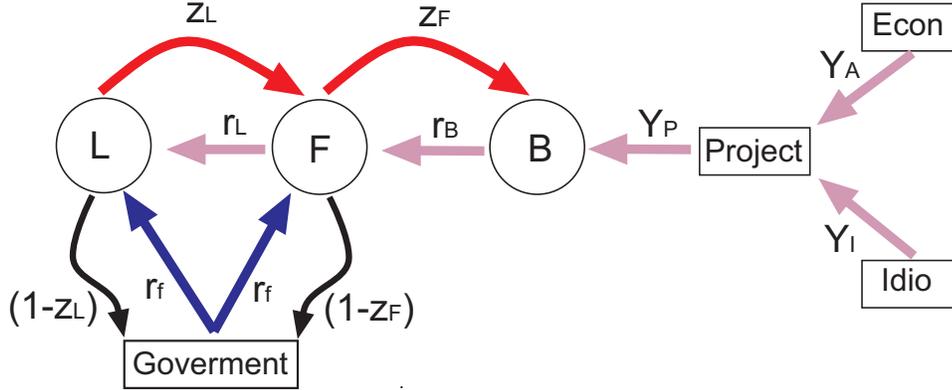
Figure 1: **The financial flows**

and his financial requirements in each period. It then derives conditions for his solvency and utilizes them to characterize the optimal project's size and B's optimal leverage conditional on the project's characteristics (its outcomes and their probabilities) and the cost of capital faced by him in periods 0 and period 1.

3.1   Real investment projects

All real investment made in period 0 are long term in the sense, that once chosen, the project's size cannot be adjusted, until returns are realized in period 2. The typical investment project yields a stochastic (net) return, $\widetilde{Y}_P$, which may be either positive or negative.[10] All real projects have the same distribution of returns, and the yields of any two different projects are correlated due to presence of the aggregate common component in $\widetilde{Y}_P$.

A project's net yield is the sum of an aggregate shock $\widetilde{Y}_A$ and of an individual idiosyncratic shock $\widetilde{Y}_I$, that is

$$\widetilde{Y}_P = \widetilde{Y}_A + \widetilde{Y}_I. \tag{1}$$

We assume, for tractability, that both the aggregate and the idiosyncratic shocks, are binomially and

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[10]Returns, whether positive or negative, are cash flows.

identically distributed. That is

$$\widetilde{Y} = \widetilde{Y}_A = \widetilde{Y}_I = \begin{cases} y, & \Pr(y) = q \\ -y, & \Pr(-y) = 1 - q \end{cases}, \tag{2}$$

where the parameter y satisfies $0 \leq y \leq 1/2$. The random variables $\widetilde{Y}_A$ and $\widetilde{Y}_I$ are statistically independent and the idiosyncratic shock, $\widetilde{Y}_I$, is independent across projects.[11] Equations (1)-(2) imply that the distribution of $\widetilde{Y}_P$ is

$$\widetilde{Y}_P = \begin{cases} 2y, & \Pr(2y) = q^2 \\ 0, & \Pr(0) = 2q(1-q) \\ -2y, & \Pr(-2y) = (1-q)^2 \end{cases}.$$

Notice that the risk of a project is a function of y: Given two projects with identical q a higher y implies a riskier project. By equations (1)-(2) the expected return of each of the component shocks $\widetilde{Y}_i$, $i = \{A, I\}$ is

$$E\widetilde{Y}_i = \overline{Y}_i = y(2q - 1), \quad i = A, I.$$

Since the project net return is the sum of $\widetilde{Y}_A$ and $\widetilde{Y}_I$, which are equal in distribution, its expected return is

$$E\widetilde{Y}_P \equiv \overline{Y}_P = 2y(2q - 1).$$

Projects must have a positive expected return to be considered; i.e. $\overline{Y}_P > 0$, which implies that $q > \frac{1}{2}$.

Assumption 1: *As of period 0 the expected return on a project is higher than the expected cost of leverage needed to carry the project to completion in period 2. That is, the distribution of the return, $\widetilde{Y}_P$, on a typical project satisfies*

$$1 \leq (1 + r_{B1})(1 + r_{B2}^e) < \left(1 + \overline{Y}_P\right),$$

where r_{B1} is the interest rate paid by the borrower in period 1 and r_{B2}^e is the interest rate he expects to pay in period 2 for refinancing in period 1 given the information set of period 0. Since the minimal

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[11]The cases $\widetilde{Y}_A = y$ and $\widetilde{Y}_A = -y$ are referred to as expansion and contraction respectively.

project size is 1 and the lowest return on a project is $-2y$ *we impose the constraint*

$$0 \le y \le \frac{1}{2}.$$

This consraint enforces limited liability by ruling out negative realizations of wealth when there is no leverage .

3.2 Borrower's financial requirements

Projects are financed by a combination of equity and of leverage supplied by financial interme-diaries to borrowers. In period 0, each borrower-entrepreneur owns one unit of equity capital. The initial financing structure (equity-1 versus leverage-L_B) is chosen by each B in period 0 along with the project's size, denoted by x. Since B's initial equity capital is 1, $x = 1 + L_B$. In each period loans by Fs to Bs are one period loans. Consequently, a B's project has to be financed by two consecutive one period loans.

In the presence of positive leverage and since projects' yields are obtained only in period 2 a B must seek refinancing in period 1. Therefore he depends on the availability and the cost of credit in period 1. If excluded from the credit market in that period he defaults and loses the entire investment project including his equity. A borrower's financial requirements in period 1 are equal to the amount needed to repay the principal, L_B, and period's 1 interest charges, $r_{B1}L_B$. Hence, *B's total financial requirements in period 1 are*

$$FR_{B1} = \underbrace{(1 + r_{B1}) L_B}_{Debt\ service}.$$

When he gets credit in period 1, B's ultimate debt service in period 2 is

$$FR_{B2} = (1 + r_{B2}) FR_{B1} = (1 + r_{B1}) (1 + r_{B2}) L_B.$$

The borrower's cost of capital for the entire project's life (from period 0 till period 2) is therefore

$$r_B \equiv (1 + r_{B1}) (1 + r_{B2}) - 1.$$

We assume that when the borrower cannot obtain refinancing in period 1 or cannot repay the debt in period 2, he defaults, the project is lost and neither the borrower nor the financial intermediary

receives any payoff. Thus, due to limited liability, the amount needed to cover losses (if any) is 0. The next section explores the borrower's solvency condition.

3.3 Borrower's solvency conditions

3.3.1 Period 0

A borrower is able to get a loan in period 0 only if the total expected payoff from his project is higher than the total debt service liability expected for period 2, that is

$$L_B \left(1 + r_B^e\right) \leq \left(1 + L_B\right) \left(1 + \overline{Y}_P\right), \tag{3}$$

where $r_B^e \equiv (1 + r_{B1})(1 + r_{B2}) - 1$ is the expected (as of period 0) cumulated interest rate factor over the lifetime of the project. Assumption 1 implies that this condition is satisfied for all non-negative leverage levels.

3.3.2 Period 1

Although all borrowers are identical exante (in period 0), they split into three groups in period 1. Those groups differ in terms of the information that becomes available to markets in that period about the realizations of their idiosyncratic shocks in period 2. In particular, it becomes known in period 1 that a fraction, $\theta_{LB} < q$, of borrowers will have $\widetilde{Y}_I = y$, a fraction $\theta_{UB} < 1 - q$ will get $\widetilde{Y}_I = -y$, and no new information is revealed in period 1 about the remaining borrowers. We refer to those three types of borrowers as Lucky borrowers (LB), Unlucky borrowers (UB) and Regular Borrowers (RB) respectively.

A borrower who decides to leverage his project in period 0 is solvent in period 1 if and only if he is able to obtain the refinancing required to maintain his project alive till period 2. Financial intermediaries will offer the required credit in period 1 if and only if the expected cash flow of the project in period 2 suffices to cover period's 1 debt service. Obviously this expected cash flow differs across borrowers' types implying that borrower of type $j = \{LB, UB, RB\}$ obtains refinancing in period 1 if and only if

$$L_B \left(1 + r_{B1}\right) \left(1 + r_{B2}\right) \leq \left(1 + L_B\right) \left(1 + E\left[\widetilde{Y}_P | I_1 \cap B_j\right]\right), \quad B_j = LB, UB, RB, \tag{4}$$

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where I_1 is the information set of period 1. Given period's 1 information

$$E\left[\widetilde{Y}_P|I_1 \cap B_j\right] = \begin{cases} 2yq, & i = LB \\ E\left[\widetilde{Y}_P \mid I_1 \cap RB\right] = 2y\left(q + q_1 - 1\right), & i = RB \\ -2y(1-q), & i = UB \end{cases} \quad (5)$$

and

$$q_1 \equiv \frac{q - \theta_{LB}}{1 - (\theta_{LB} + \theta_{UB})} \equiv \frac{q - \theta_{LB}}{1 - \theta} \quad (6)$$

is the probability that a regular borrower will get a good draw on the idiosyncratic shock, $\widetilde{Y}_I$, given the information available in period 1.[12]

Assumption 2: *The expected net return of a RB on a real project conditional on the information in period 1 satisfies* $E\left[\widetilde{Y}_P \mid I_1 \cap RB\right] > (1 + r_{B1})\left(1 + r_{B2}^e\right) \equiv r_B^e.$

Assumption 2 is basically an extension of Assumption 1 from period 0 to period 1. Together those assumptions requires that, given r_B^e, the expected net return perceived by a regular borrower is larger than the expected cost of leverage given the information of both periods 0 and 1. The following Lemma identifies solvency conditions in period 1 for the three types of borrowers

Lemma 1:

(i) Regular borrowers are solvent in period 1 at any level of leverage, L_B, if $r_{B2} = r_{B2}^e$.

(ii) Lucky borrowers are solvent in period 1 at any level of leverage if $2yq > E\left[\widetilde{Y}_P \mid I_1\right]$.[13]

(iii) Unlucky borrowers are solvent in period 1 if and only if $L_B \leq \frac{1 - 2y(1-q)}{2y(1-q) + r_B^e} \equiv L_B^1$

Note that since $\frac{1}{2} < q \leq 1$ the critical level of leverage, L_B^1, is positive. The lower this critical level the wider the range of period's 0 debt for which there is a non-zero probability that the unlucky borrower defaults in period 1. When the expected cost of capital, r_B^e, increases the critical level, L_B^1, decreases, implying that, the higher the cost of capital, the wider is the range of leverages at which a borrower might default in period 1. Increasing risk (measured in terms of returns' variance) has a similar effect since it decreases L_B^1.[14] On the other hand, when the probability of good returns

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[12]The reason this probability differs between periods 0 and 1 is that the realizations of $\widetilde{Y}_I$ become known with certainty for a fraction, θ, of borrowers in period 1.

[13]Overly strong jointly sufficient conditions for this requirement are $\theta_{LB} = \theta_{UB}$ and $q > 1 - q..$

[14]It is easily checked that increasing y, while keeping the probabilities of good and bad outcomes (q and $(1 - q)$)

increases (i.e q increases), the critical leverage, L_B^1, increases widening the range of leverages for which the probability of default in period 1 is zero.

3.3.3 Period 2

A borrower is solvent in period 2 if the payoff from his project suffices to cover his debt obligation, that is

$$L_B \left(1 + r_{B1}\right) \left(1 + r_{B2}\right) \leq \left(1 + L_B\right) \left(1 + \widetilde{Y}_P\right). \tag{7}$$

Straightforward algebra shows that this is equivalent to the requirement that ultimate wealth, $W_B(.)$, is non negative

$$W_B(L_B, \widetilde{Y}_P) = 1 + \widetilde{Y}_P + \left(\widetilde{Y}_P - r_B\right) L_B \geq 0. \tag{8}$$

When the final project's payoff does no suffice to cover the principal and interest rate payments the borrower defaults and loses the entire project including his initial equity. Due to limited liability the net return on the project in this case is $\widetilde{Y}_P = -1$ and the financial intermediary who own the debt receives nothing.

Lemma 1 has shown that when period's 0 leverage is higher than some critical value the borrower is exposed to default risk in period 1. In addition positive leverage also exposes borrowers to default risk in period 2. The following Lemma identifies regular borrower's solvency conditions in period 2 for various realizations of total returns.

Lemma 2: *If a regular borrower's ultimate return is:*

(i) $\widetilde{Y}_P = -2y$ he is solvent in period 2 if and only if $L_B \leq \frac{1-2y}{r_B+2y} \equiv L_B^L$,

(ii) $\widetilde{Y}_P = 0$ he is solvent in period 2 if and only if $L_B \leq \frac{1}{r_B} \equiv L_B^H$,

(iii) $\widetilde{Y}_P = 2y$ he is solvent in period 2 for any level of leverage.

Lemma 3: Given $r_{B2} = r_{B2}^e$,

(i) If an unlucky borrower (UB) has chosen L_B^1 in period 0 he is solvent in period 1.

(ii) An UB that has chosen L_B^1 is also solvent in period 2 if and only if there is an aggregae expansion

(iii) A lucky borrower is always solvent in period 2.

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unchanged, increases the project's variance without changing its expected return.

Next, we analyze the probabilities of default in periods 1 and 2 as functions of the leverage chosen in period 0. Since payoffs are discrete those relations take the form of step functions.

Proposition 1: *Provided $r_{B2} = r^e_{B2}$, the ex-ante probabilities of default in period 1 and in period 2 (as viewed from he vantage point of period 0) are step functions of the leverage chosen in period 0. The precise probabilities of defaults are:*

L_B	$\Pr(D_1)$	$\Pr(D_2)$	$\Pr(D) = \Pr(D_1) + [1 - \Pr(D_1)]\Pr(D_2)$
$L_B \leq L_B^L$	0	0	0
$L_B^L < L_B \leq L_B^1$	0	$(1-q)^2$	$(1-q)^2$
$L_B^1 < L_B \leq L_B^H$	θ_{UB}	$(1-q)^2$	$\theta_{UB} + (1-\theta)(1-q)^2$
$L_B^H < L_B$	θ_{UB}	$1-q^2$	$\theta_{UB} + (1-\theta_{UB})(1-q^2) + ((1-\theta)2q + \theta_L)(1-q)$

,

where $\Pr(D_1)$ and $\Pr(D_2)$ stand for default probabilities in period 1 and 2 respectively.

3.4   Borrower's optimization

Not surprisingly the individual borrower faces a tradeoff between expected payoff and default probability. In the large, by raising leverage, he raises the expected value of terminal equity but also the chances of default. By Proposition 1 the exante probability of default is a step function of leverage. This implies that the optimal level of leverage (and by implication also the optimal project's size) must coincide with one of the four leverage levels at the jump points of the probability of default function. The reason is that, once leverage is extended beyond a given jump point the probability of default remains constant as long as leverage is not pushed beyond the next jump point. Within such an interval, raising leverage raises the expected payoff without raising the probability of default.

Hence, once leverage is raised beyond a given jump point, it is individually optimal to push it (at least) all the way till just a tiny bit before the probability function's next jump point. It follows that, from the vantage point of period 0, the optimal level of leverage is either zero or one of the

following three leverage levels:

$$L_B^L = \frac{1 - 2y}{r_B^e + 2y};$$

$$L_B^1 = \frac{1 - 2y(1-q)}{2y(1-q) + r_B^e}; \qquad (9)$$

$$L_B^H = \frac{1}{r_B^e}.$$

The borrower's utility function is piecewise linear with a penalty in the event of default. In particular utility is linear in wealth as long as the borrower is solvent. When insolvent the borrower is subject to a penalty that increases with the magnitude of leverage he defaults on. Formally

$$u(W_B, L_B) = \begin{cases} W_B \geq 0, & \text{Solvency} \\ P_B L_B, & \text{Insolvency} \end{cases}, \qquad (10)$$

where W_B is his period's 2 terminal wealth after servicing all debts and P_B is a fixed default penalty per unpaid leverage dollar in states of insolvency. Hence, the borrower's expected utility is

$$V(L_B) \equiv Eu(W_B(\cdot), L_B)$$

$$= [1 - \Pr(D \mid L_B)] E[W_B(\cdot) \mid W_B(\cdot) \geq 0] - \Pr(D \mid L_B) P_B L_B. \qquad (11)$$

Using Proposition 1 and the definition of $W_B(\cdot)$ in equations (8) and (9) establishes that $B's$ expected utilities at each of the five candidates for optimal leverage (four discussed above plus any level of leverage $L_B^m > L_B^H$) are given by

$$V(L_B = 0) = q^2(1 + 2y) + 2q(1-q) + (1-q)^2(1-2y);$$

$$V(L_B^L) = q^2\left[1 + 2y + (2y - r_B^e)L_B^L\right] + 2q(1-q)\left[1 - r_B^e L_B^L\right];$$

$$V(L_B^1) = q^2\left[1 + 2y + (2y - r_B^e)L_B^1\right] + 2q(1-q)\left[1 - r_B^e L_B^1\right] - (1-q)^2 P_B L_B^1; \qquad (12)$$

$$V(L_B^H) = \left(\theta_{LB}q + (1-\theta)q^2\right)\left[1 + 2y + (2y - r_B^e)L_B^H\right] - \Pr\left[D \mid L_B^H\right] P_B L_B^H;$$

$$V(L_B^m) = \left(\theta_{LB}q + (1-\theta)q^2\right)\left[1 + 2y + (2y - r_B^e)L_B^m\right] - \Pr\left[D \mid L_B^m\right] P_B L_B^m,$$

where

$$\Pr\left[D \mid L_B^H\right] \equiv \left[\theta_{UB} + (1-\theta)(1-q^2)\right],$$

$$\Pr\left[D \mid L_B^m\right] \equiv \Pr\left[D \mid L_B^H\right] + ((1-\theta)2q + \theta_{LB})(1-q).$$

Let L_B^* be the optimal level of leverage. The following proposition presents (overly restrictive) sufficient condition for $L_B^* = L_B^H$.

Proposition 2: *Provided*

$$P_B > \frac{(\theta_{LB}q + (1-\theta)q^2)(2y - r_B^e)}{((1-\theta)2q + \theta_{LB})(1-q)} \equiv P_B^c,$$

there exists a dense set of values for the vector of parameters $(q, \theta_{LB}, \theta_{UB})$, such that $1-q$, $q-\theta_{LB}$ and θ_{UB} are all strictly positive but small, for which the borrower's optimal level of leverage is L_B^H.

The broad intuition underlying this proposition can be appreciated by starting with the particular case in which the unit penalty, P_B, for default is zero. In this case, when the chances of good draws at the individual level are high (q and θ_{LB} are large) and the likelihood that the borrower will be unlucky in period 1 is low (θ_{UB} is small), expected utility is monotonically increasing in leverage. As a matter of fact, given the full linearity of the utility function in the absence of a penalty, the borrower's optimal level of leverage is infinite in this case. However, in the presence of a sufficiently large default penalty extending leverage beyond L_B^H is not individually optimal because of the increase in the risk that the penalty will be triggered once leverage crosses the L_B^H threshold.

In a broad sense the conditions in the Proposition 2 are analogous to the borrower's second order condition (SOC) when the penalties from default rise continuously with leverage. In the continuous case the SOC assures that, as leverage goes up, the favorable marginal impact of higher leverage on return in good states diminishes in comparison to the unfavorable gradual increase in the default penalty. Similarly, the conditions in Proposition 2 assure that, as leverage rises, the marginal detrimental impact of the default penalty becomes more important relatively to the marginal favorable effect on likely profits.

4   Financial intermediaries (Fs)

For reasons that will become apparent later it is convenient to open this section with a forward look at the relation between various equilibrium rates of interest.

4.1 A forward look at general equilibrium

The following proposition establishes general equilibrium relations between equilibrium interest rates, r_B, r_F and r_L.

Proposition 3: *In a general equilibrium with risk aversion on the part of borrowers, financial intermediaries and lenders, and positive levels of leverage in both the real and the financial sectors, the following inequalities hold*

$$r_f \leq r_L < r_B.$$

4.2 The typical financial intermediary

There is a large number of financial intermediaries (Fs) each of which possesses one unit of core funds consisting of a combination of equity and of long term (two periods) debt. A typical F can also raise **short term** (one period) funds from lenders.[15] Since the focus of this analysis is on changes in the availability of short term credit in the face of new information, the amount of short term leverage assumed by a typical F is determined endogenously while the sum of equity and of long term debt is taken to be exogenous.

Total financial resources of a typical F consist of the core funds and of short term leverage, L_F. The financial intermediary diversifies his total resources between the risk free asset whose rate, r_f, is a policy instrument, and a risky, not fully diversified, portfolio of loans to borrowers.[16] For reasons of tractability, each F lends to only two borrowers. The fraction of resources invested in the risky loan portfolio to Bs is denoted z_F. Let W_F be the intermediary's terminal wealth after debt service in each period. F is solvent or insolvent in each period depending on whether terminal wealth is non negative or strictly negative. When solvent, F's utility is described by a CRRA utility function with a coefficient, $\delta > 0$, of risk aversion that is close to, but not quite equal to, 0. This specification implies that F is almost, but not strictly, risk neutral. When insolvent, the typical intermediary

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[15]For instance through various deposits including certificates of deposit (CDs).

[16]By contrast, as shown in the next section, the risky portfolio of suppliers of funds to Fs (lenders) is fully diversified.

experiences a (per unit of leverage) penalty, P_F. Formally[17]

$$u\left(W_F\right) = \begin{cases} \frac{[W_F]^{1-\delta}}{1-\delta}, & \text{when } W_F \geq 0 \\ \\ -P_F L_F, & \text{when } W_F < 0. \end{cases} \tag{13}$$

4.3   Distribution of returns, solvency and optimization

Total return to a financial intermediary depends on the performance of the two borrowers to whom he lends. Since borrowers are identical exante, the optimal risky portfolio of an F consists of a fifty-fifty split between loans to his two debtors. If both borrowers are solvent both of them pay the full face value, of the gross debt service and the payoff (from one unit) to F is $1 + r_B$. If both of them default F gets a payoff of 0 on his risky portfolio. If one borrower is solvent and the other defaults F gets the payoff $\frac{1}{2}(1 + r_B)$. Obviously, the probabilities associated with each of those three payoffs depend on the probabilities of defaults of borrowers and differ between periods 0 and 1. From the vantage point of period zero, the probability a single B defaults in period 1 , $\Pr(D_1)$, is given in Proposition 1. Since $L_B^* = L_B^H$, the probability that a B is insolvent in period 1 is θ_{UB}. Since the probability of being unlucky of any borrower is statistically independent of this probability for any other borrower, the distribution of payoffs faced by a typical intermediary in period 0 is given by the following table.[18]

		Period 1	**Period 2**	
State	Payoff ($\widetilde{R}_B$)	**Probability**	**Probability**	
Both Bs are solvent	$1 + r_B$	$(1 - \theta_{UB})^2 \equiv \gamma_{11}$	$q(1 - q_1^2) + q_1^2 \equiv \gamma_{12}$	(14)
Exactly one B is solvent	$\frac{1}{2}(1 + r_B)$	$2(1 - \theta_{UB})\theta_{UB} \equiv \gamma_{21}$	$2(1 - q)q_1(1 - q_1) \equiv \gamma_{22}$	
Both Bs are insolvent	0	$\theta_{UB}^2 \equiv \gamma_{31}$	$(1 - q)(1 - q_1)^2 \equiv \gamma_{32}$	

Recall that q is the probability of a positive (aggregate or idiosyncratic) shock and q_1, defined in Equation (6), is the probability of a positive idiosyncratic shock conditional on the project type that is revealed in period 1. The last column shows the probability distribution of period's 2 payoffs from

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[17]In spite of the fact that utility functions differ across the three types of agents we use the symbol $u(\cdot)$ to stand for all of them in order to economize on notation. In each case the identity of the player should be evident from the context.

[18]The notation γ_{bt} stands for probability in time t conditional on realized borrowers' types, b.

loans to regular borrowers as perceived by F's in period 1.

The wealth of a typical F at the end of each period is

$$\widetilde{W}_F(\widetilde{R}_B, L_F) = (1 + L_F)\left[z_F\widetilde{R}_B + (1 - z_F)(1 + r_f)\right] - (1 + r_L)L_F, \tag{15}$$

where the distributions of $\widetilde{R}_B$ is given in Equation (14) and r_L is the interest rate paid by a F on its short term obligations. A representative F chooses his leverage, L_F, and the fraction, z_F, of resources invested in the risky loan portfolio so as to maximize $Eu(\widetilde{W}_F)$ in each of periods 0 and 1. The following proposition presents a preliminary characterization of F's optimal policy.

Proposition 4: *Let $r_f < r_L$. Then at an optimum with positive leverage, F invests all his resources in risky loans to Bs.*

4.3.1 F's solvency condition

Proposition 4 and Equation (15) imply that F is solvent if and only if

$$\widetilde{W}_F(\widetilde{R}_B, L_F) = (1 + L_F)\widetilde{R}_B - (1 + r_L)L_F = \widetilde{R}_B + (\widetilde{R}_B - r_L)L_F \geq 0. \tag{16}$$

Since $r_B > r_L$, F is solvent for any level of leverage, L_F, when both of his borrowers are solvent, so that $\widetilde{R}_B = 1 + r_B$. In the other two cases F is solvent only if L_F is sufficiently small. The precise solvency conditions are:

$$
\begin{array}{lll}
L_F \leq \frac{1+r_B}{1+2r_L-r_B} \equiv L_F^c & \text{when} & \widetilde{R}_B = \frac{1}{2}(1 + r_B) \\
L_F = 0 & \text{when} & \widetilde{R}_B = 0
\end{array}
\tag{17}
$$

Equations (14) and (17) imply that F's probability of default is an increasing step function of F's leverage and that the precise functions for periods 1 and 2 are

$$
\begin{array}{cc}
\mathbf{L}_F & \Pr\left[D_t\right], \ \mathbf{t = 1, 2} \\
0 & 0 \\
L_F \leq L_F^c & \gamma_{1t} \\
L_F > L_F^c & \gamma_{2t} + \gamma_{3t}
\end{array}
\tag{18}
$$

Proposition 5: *Provided:*

(i) δ is sufficiently small,

(ii) $\gamma_{1t}(r_{Bt} - r_{Lt}) - (1 - \gamma_{1t})P_F > 0$,

(iii) $[\gamma_{1t}(r_{Bt} - r_{Lt}) - (1 - \gamma_{1t})P_F](L_{Ft}^* - L_F^c) > \gamma_{2t}P_F L_F$,

(iv) $(\gamma_{2t}/\gamma_{1t})$ is sufficiently small,

F's optimal leverage is

$$L_{Ft}^* = \left(\frac{\gamma_{1t}}{1 - \gamma_{1t}}\right)^{\frac{1}{\delta}} (r_{Bt} - r_{Lt})^{\frac{1-\delta}{\delta}} - \frac{1 + r_{Bt}}{r_{Bt} - r_{Lt}}. \tag{19}$$

Here L_{Ft}^*, $t = \{0, 1\}$ is the financial intermediary's optimal short term leverage in periods 0 and 1.

The following proposition formulates the financial intermediary's solvency condition. If solvent a financial intermediary pays the full debt service to lenders. Otherwise he defaults and pays nothing.

Proposition 6: *Provided δ is sufficiently small the financial intermediary is solvent if and only if the two borrowers to whom he has lent are solvent.*

Proposition 6 implies that (since $L_{Ft}^* > L_F^c$) overly restrictive sufficient conditions for the two requirement in Proposition 5 are that γ_{1t} is sufficiently large and/or P_F sufficiently small. Next, we characterize F's optimal leverage.

Proposition 7: *The optimal leverage of a typical financial intermediary is higher*

(i) the lower are the intermediary's risk aversion, δ, and the default penalty, P_F,

(ii) the lower the cost of borrowing, r_{Lt},

(iii) the higher the probability, γ_{1t}, that the intermediary remains solvent,

(iv) the higher the interest rate, r_{Bt}, paid by borrowers.

5   The representative lender and bailout policy

Through pension or mutual funds the representative lender (L) splits his equity between a fully diversified portfolio of loans to financial intermediaries and the risk free asset.[19] Since, exante, all Fs have identical distributions of returns the optimal shares of loans to different Fs are all equal. The fraction invested in the risky loan portfolio to Fs is denoted z_L. The typical lender possesses

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[19]The lender is representative in the sense that all lenders are identical.

mean-variance (or Constant Absolute Risk Aversion - CARA)[20] preferences

$$u\left(W_L\right) = -\tfrac{1}{\alpha}e^{-\alpha W_L}, \qquad \alpha \geq 0, \tag{20}$$

where W_L is his terminal wealth in each period and α characterizes the degree of constant absolute risk aversion.

5.1 Perceived government's bailout policy

Government may repay the gross debt owed to lenders by defaulting Fs. The perceived probability that the debt service of a defaulting F is paid by government (a bailout) is denoted by p. The likelihood of bailout is independent across Fs debt. In case of bailout a lender receives the full debt service, r_L. In the presence of risk but no bailout uncertainty p is unique. We use the Gilboa Schmeidler's (1989) multiple priors framework to formalize Knightian uncertainty and aversion to it[21] Accordingly, in the presence of bailout uncertainty perceptions include the convex set of all possible binomial distributions characterized by p's that the lenders believe to have positive mass (i.e. are considered as plausible). The lowest value of p in the set is denoted by π. As will become clear below this is also the worst plausible prior from a typical lender's point of view.

The degree of uncertainty and of aversion to it is determined by the "size" of the set of possible priors. That is, when circumstances become more uncertain (ambiguous), or when ambiguity aversion increases, the set of possible priors expands to include priors that previously were considered implausible. Consequently, provided some of the set enlargement is toward lower p's, the worst prior, π, is revised downward.

5.2 Representative lender's returns and optimization

5.2.1 Period 0

On one period loans taken in period 0 borrowers face only idiosyncratic risk because only individually unlucky borrowers default in period 1, implying that financial intermediaries who lend to them also face only idiosyncratic risk in that period. By contrast, since they fully diversify their loans across

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[20]See Sargent (1987) pages 154-155.

[21]Knightian uncertainty and aversion to it are also known as "ambiguity" and "ambiguity aversion" in modern decision theory.

intermediaries, lenders face no risk at all in period 0. Consequently, and since lenders know from Equation (14) that only a fraction, $(1 - \theta_{UB})^2$, of Fs will be solvent in period 1, equilibrium r_{L1} includes a compensation for the average fraction of unpaid debt but no compensation for variability of this fraction, since this variability equals zero due to full diversification. Hence,

$$1 + r_{L1} = \frac{1 + r_{f1}}{1 - (1 - \pi)\left[1 - (1 - \theta_{UB})^2\right]} \tag{21}$$

where $(1 - \pi)\left[1 - (1 - \theta_{UB})^2\right]$ is the probability that a lender loses the investment in a loan to a single intermediary. It is obtained as the product of the probabilities of the two following independent events: "the intermediary defaults" and "government does not reimburse the delinquent debt to the lender". As a consequence, in period 0, lenders are indifferent between investing in the standard risk free asset at rate r_{f1} and between investing in loans to Fs.

5.2.2 Period 1

By contrast, in period 1, lenders face risk in loans to Fs in spite of their fully diversified portfolios. The reason is that the returns to lenders from loans to different Fs are correlated due to the common shock, $\widetilde{R}_A$, in the return to real investments of borrowers. As explained in the previous section a financial intermediary either pays his debt in full to lenders or fully defaults. When F defaults on the debt service, government may or may not step in and pay the delinquent debt service to a lender. Consequently the lender faces a binomial distribution of returns from lending to an individual F – he either gets the full debt service, $1 + r_L$, (from F or from government) or 0. Although the bailout policy of government does not affect the binomial nature of the payoffs from a single F, it **does alter their distribution**. Since the risky portfolio of L's contains a large number of such binomially distributed loans the risky portfolio of Ls is normally distributed with a mean and a variance that depend on both economic (q) and political (p) uncertainties. Details appear in the following proposition.

Proposition 8: *For a given p period's 2 payoff to a lender on his fully diversified portfolio of loans,* $\{\widetilde{R}_{L2}\}$, *is normally distributed with mean*

$$E\left(\{\widetilde{R}_{L2}\}\right) = [p + \gamma_{12}(1 - p)](1 + r_{L2}) = \left[p + \left[q(1 - q_1^2) + q_1^2\right](1 - p)\right](1 + r_{L2}) \equiv \lambda_1 1 + r_{L2}) \tag{22}$$

and variance[22]

$$Var\left(\{\widetilde{R}_{L2}\}\right) = (1-p)^2\,(1-q)\,(1-q_1)^2\left(q+2qq_1+q_1^2\right)(1+r_{L2})^2 \equiv \lambda_2(1+r_{L2})^2, \quad (23)$$

where

$$q_1 \equiv \frac{q-\theta_{LB}}{1-(\theta_{LB}+\theta_{UB})}, \quad (24)$$

and $\{\widetilde{R}_{L2}\}$ stands for the set of possible returns from loans to F's.

A representative L chooses the fraction, z_L, of resources invested in the risky loan portfolio to Fs so as to maximize[23]

$$E\left(\widetilde{W}_L(z_L)\right) = -\frac{1}{\alpha}E\left(e^{-\alpha\widetilde{W}_L(z_L)}\right) \quad (25)$$

in each period, where

$$\widetilde{W}_L(z_L) = z_L\widetilde{r}_L + (1-z_L)r_f. \quad (26)$$

Proposition 9: *At an individual optimum, a lender allocates the fraction*

$$z_L^*\left(\pi, R_L, q, q_1\right) \cong \frac{E\left(\{\widetilde{R}_L\}\right)-(1+r_f)}{\alpha Var\left(\{\widetilde{R}_L\}\right)} \quad (27)$$

of each single \$ to the diversified risky portfolio of loans to Fs.[24,25]

5.3   Partial equilibrium comparative statics

We now investigate the impact of less generous bailouts on the size of lenders' risky portfolios and the impact of ambiguity aversion in partial equilibrium. In the absence of bailout uncertainty government's bailout policy is characterized by a unique perceived probability, p, that government will pay the debt of delinquent F's to L's. A more generous (towards L's) bailout policy is characterized by a higher p and a less generous bailout policy by a lower p. By changing the distribution of $\widetilde{r}_L$ the value of p affects both the mean and the variance of lenders' risky portfolios.

Proposition 10: *Holding r_{L2} constant a perceived less generous bailout policy (lower p)*

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[22]The variance is scaled by the term $(1+r_{L2})^2$ for reasons of space.

[23]The right hand side of Equation (25) is obtained by using a typical lender's utility function in equation (20).

[24]This approximation is accurate for a small risk premium, $E\left(\{\widetilde{r}_L\}\right)-r_f$.

[25]An identical allocation, $z_L^*\left(\pi, R_L, q, q_1\right)$, is obtained for constant relative risk aversion (CRRA) preferences (see Merton 1971, 1973).

(i) reduces the mean return on the portfolio of loans from lenders to financial intermediaries,

(ii) raises the covariance between any two loans in the (fully diversified) portfolio, and therefore, the portfolio's variance,

(iii) Both changes reinforce each other in inducing a "flight to safety" by lenders.

In the presence of uncertainty about p, and since they are averse to ambiguity in the Gilboa-Schmeidler (1989) sense, lenders behave as if the probability of bailout is the lowest within the set of p's with positive mass (denoted π).[26],[27] Stated differently, they choose the fraction of their portfolio invested in risky loans to Fs so as to maximize expected utility under the assumption that bailout probability is π. The operational consequence of such behavior is that p should be replaced with π in propositions 8 and 10. We remind the reader that this behavioral mode reflects both uncertainty about the correct bailout distribution as well as aversion to this uncertainty.

Propositions 10 implies that higher bailout uncertainty and/or aversion has two effects: Not surprisingly it lowers the expected return from the risky loan portfolio of lenders. More surprisingly, but not less importantly, it raises the correlation between loans in the portfolio which implies in turn higher variances in lenders portfolios. This result appears surprising at first blush since intuition may lead one to conclude that an increase in bailout probability, by decreasing the likelihood of default, will increase the correlation between loans' returns in the portfolio. But this intuition is mistaken. The reason is that the correlation originates uniquely from the aggregate shock whose impact operates only through the fraction of loans in the portfolio that are no bailed out. Since the impact of this fraction on the overall correlation diminishes as more intermediaries are bailed out the variance goes down. Consequently, in the limit, when bailouts are almost certain, this variance tends to zero.

The aggregate covariance between all pairs of loans in the fully diversified portfolio is one measure of systemic risk in financial markets. Hence, Part (ii) of Proposition 10 implies that an increase in bailout uncertainty raises systemic risk. Furthermore, recalling (from the introduction) that

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[26]For simplicity we use the Gilboa-Schmeidler (1989) model to capture uncertainty, however other model of uncertainty can be incorporated into our model; Klibanoff, Marinacci and Mukerji (2005) or Izhakian and Izhakian (2009a, 2009b), for example.

[27]Lowest in the sense that under this probability distribution expected utility attains its minimal value.

expansion of the set of multiple priors toward lower bailout probabilities is likely to reflect a persistent increase in ambiguity aversion following traumatic events, Part (ii) of Proposition 10 implies that **systemic risks depend also on preferences**.

Proposition 10 implies that, when due to an increase in bailout uncertainty and/or aversion to it π decreases, lenders reduce the share of funds supplied to financial intermediaries. This conclusion plays an important role in the following general equilibrium sections.

Proposition 11: *Provided $1 + 2r_f \geq r_L$, L's optimal allocation to risky loans, $z_L^*(\pi, r_L, q, q_1)$ is increasing in r_L.*

It is easy to check that the sufficient condition in the proposition is satisfied for practically all normal levels of interest rates.

6 General equilibrium of the financial system

Given expectations about the future, general equilibrium of the financial system is characterized by two market clearing conditions. One for credit from Ls to Fs and the other for credit from Fs to Bs. These two conditions simultaneously determine r_B and r_L in each period. In period 1, expectations about the future only involve realizations of period's 2 returns to borrowers. As a consequence, the formulation of this equilibrium is relatively simple. But in period 0 they also involve expectations about period's 1 market clearing values of r_B and r_L in period 1 (r_{B2}^e and r_{L2}^e). Those expectations are assumed to be model consistent in the sense that, in period 0, financial market participants use the information available in that period along with their knowledge of the fact that period's 1 rates will be determined by market clearing to derive r_{B2}^e and r_{L2}^e.

6.1 General equilibrium in period 0

Proposition 1 implies that a borrower is insolvent in period 1 only if he is unlucky, implying (from the discussion in Section 4) that a financial intermediary defaults in period 1 if and only if at least one of his borrowers is unlucky. Being unlucky is related to B's and Fs individual fortunes rather than to the aggregate shock. Lenders are exposed to aggregate shocks and idiosyncratic shocks; aggregate shocks are relevant only in period 2 while idiosyncratic shocks are fully diversified. Hence,

since lenders are fully diversified and lend to Fs for only one period, they do not face any risk in lending to them in period 0. In particular they know for sure (from Equation (14)) that a fraction $(1 - (1 - \theta_{UB})^2)$ of intermediaries will default in period 1. Hence, they demand a compensation only for this known with certainty fraction of defaults. Consequently, in period 0, r_{L1} is determined exogenously by the condition

$$1 + r_{L1} = \frac{1 + r_{f1}}{1 - (1 - p)\left[1 - (1 - \theta_{UB})^2\right]}. \tag{28}$$

Actual period's 0 equilibrium conditions in the markets for loans from Ls to Fs and from Fs to Bs are given respectively by

$$M_F L_F^*(r_{B1}, r_{L1}) = M_L z_{L1}, \tag{29}$$

$$L_B^*(r_{B1}, r_{B2}^e) = \frac{M_B}{(1 + r_{B1})(1 + r_{B2}^e) - 1} = M_F \left(1 + L_F^*(r_{B1}, r_{L1})\right). \tag{30}$$

They determine z_{L1} and r_{B1} as functions of r_{L1} and r_{B2}^e. Since $L_B^*(r_{B1}, r_{B2}^e)$ also depends on period's 0 expectation of the cost of funds to borrowers, r_{B2}^e, in the subsequent period a full characterization of period's 0 equilibrium requires additional conditions for the determination of r_{B2}^e. Those conditions are provided by the hypothesis that, in period 0, agents form their perceptions about r_{B2}^e and r_{L2}^e by utilizing period's 1 expected market clearing conditions given their period's 0 information. Expected period's 1 equilibrium conditions in the markets for loans from Ls to Fs and from Fs to Bs are given respectively by

$$(1 - \theta_{UB})^2 M_F L_F^*(r_{B2}^e, r_{L2}^e) = (1 + r_{f1}) M_L z_L^*(\pi_0, r_{L2}^e, q, q_1), \tag{31}$$

$$\frac{M_B}{(1 + r_{B1})(1 + r_{B2}^e) - 1} = (1 - \theta_{UB}) \left[\begin{array}{l} 1 + r_{B1} + (r_{B1} - r_{L1})L_F^*(r_{B1}, r_{L1}) \\ + L_F^*(r_{B2}^e, r_{L2}^e) \end{array} \right]. \tag{32}$$

The two period's 0 market clearing conditions and the two clearing conditions expected for period 1 jointly determine r_{B1}, r_{B2}^e, r_{L2}^e and z_{L1}. This system is recursive since the last three equations simultaneously determine the first three variables leaving the first equation for the determination of z_{L1}.

6.2　General equilibrium in period 1

The actual period's 1 market clearing conditions in the markets for loans from Ls to Fs and from Fs to Bs are given respectively by

$$(1 - \theta_{UB})^2 M_F L_F^*(r_{B2}, r_{L2}) = (1 + r_{f1}) M_L z_L^*(\pi_1, r_{L2}, q, q_1), \tag{33}$$

$$\frac{M_B}{(1 + r_{B1})(1 + r_{B2}^e) - 1} = (1 - \theta_{UB}) \left[\begin{array}{l} 1 + r_{B1} + (r_{B1} - r_{L1}) L_F^*(r_{B1}, r_{L1}) \\ \\ + L_F^*(r_{B2}, r_{L2}) \end{array} \right]. \tag{34}$$

Those equilibrium conditions determine the *actual* interest rates, r_{B2} and r_{L2} in period 1 for predetermined, values of r_{B1}, r_{B2}^e, r_{L1} and of π_1. Comparing $z_L^*(\pi_0, r_{L2}^e, q, q_1)$ from Equation (31) with $z_L^*(\pi_1, r_{L2}, q, q_1)$ from Equation (33) we note that the first two arguments of those functions differ. Not surprisingly the first, which refers to the expected equilibrium, features r_{L2}^e, while the second, that refers to the actual equilibrium, features r_{L2}. Importantly, the effective bailout probabilities, π_0 and π_1, differ across the expected and the actual period's 1 equilibria. This (at this stage) notational difference is introduced in anticipation of the discussion in the next section that introduces an unanticipated increase in bailout uncertainty.

7　The impact of bailout uncertainty

This section considers the impact of an unanticipated increase in bailout uncertainty on financial markets in period 1. Recall first that π_0 is the lowest bailout probability in the multiple priors set as perceived in period 0 for **both** periods 0 and 1.[28] The fact that, as of period 0, financial market participants do not expect this probability to change in period 1 is reflected in the formulation of the expected period's 1 general equilibrium conditions for period 0 (equations (29) through (32)).

Suppose now that, following a major indication of a shift in government's bailout policy—like not rescuing Lehman—bailout uncertainty and/or aversion to it increase. In particular, the lowest perceived bailout probability with positive mass decrease from π_0 to π_1. Proposition 10 implies that, holding r_{L2} constant, this change reduces the supply of funds to Fs by Ls. Application of

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[28]More generally π_t stands for the lowest belief in period t about the probability of governmental bailout in all relevant future periods.

comparative statics methods to period's 1 equilibrium conditions shows that this change triggers a general equilibrium increases in both r_{L2} and r_{B2} above their expected counterparts – r_{L2}^e and r_{B2}^e. Those increases raise the fraction of defaulting borrowers in period 2 and may, under some circumstances, lead to a total drying up of credit in period 1. The string of propositions in this section provides a precise formulation of these and related results

Let

$$\frac{\partial L_F^*}{\partial r_B}, \quad \frac{\partial L_F^*}{\partial r_L} \tag{35}$$

be respectively the responses of $F's$ optimal leverage to changes in r_{B2} and in r_{L2} and let

$$\frac{\partial z_{L2}^*}{\partial r_L}, \quad \frac{\partial z_{L2}^*}{\partial \pi} \tag{36}$$

be the responses of $L's$ optimal share of investments in the risky portfolio to changes in r_{L2} and in π.

Proposition 12: *Given propositions 7, 10, and 11 an increase in uncertainty about governmental bailout policy and/or an increase in aversion to it (and thus a decrease in the effective probability of bailout from π_0 to π_1) leads to:*

(i) A flight to quality by lenders (z_{L2} goes down)

(ii) An increases in r_{L2} and r_{B2} above r_{L2}^e and r_{B2}^e respectively;

*(iii) When the increase in r_{B2} is such that $r_{B2} > r_{B2}^e + 2y(q + q_1 - 1)(1 + r_{B2}^e)$, period's 1 credit is denied to **both regular** and unlucky borrowers, so both types of borrowers default in period 1;*

*(iv) A sufficiently large increase in r_{B2} beyond $r_{B2}^e + 2y(q + q_1 - 1)(1 + r_{B2}^e)$, triggers a total "financial arrest" in period's 1 credit to Bs in the sense that credit is denied to **all** borrowers.*[29]

The comparative statics impacts in Proposition 12 accord well with the flight to quality and the general increase in the cost of debt observed following the downfall of Lehman Brothers. They are consistent with the view that much of the financial market panic, and the associated arrest of financial markets, in the aftermath of this collapse was due to increases in uncertainty and uncertainty aversion about the willingness of the US government to use public funds to compensate creditors'

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[29]This term has recently been suggested by Ricardo Caballero (2010) .

for losses due to defaulting financial intermediaries.

Following the outbreak of the crisis one of the expost explanations offered for the fact that so many market participants failed to see it coming was that they underestimated the systemic risks due to the correlation between the returns on different assets. The framework of this paper suggests that underestimation of this correlation prior to the crisis may be due to the fact that, by proposition 10 and 12, optimistic bailout expectations are associated with a low perceived correlation and therefore a low variance in lenders portofolios. On this view underestimation of the covariance between returns prior to the decision not to bailout Lehman was rational in view of the optimistic beliefs held at the time about government bailout policy. But after Lehman's collapse uncertainty about bailout policy and the aversion to this uncertainty increased. This led to a more pessimistic evaluation of the likelihood of bailouts raising the covariance between the returns on loans to financial intermediaries, and with it the variance of lenders' portofolios. The implication of this point of view is that the alleged underestimation of systemic risks prior to Lehman's collapse might have been rational in view of the fact that the Fed had neutralized the impact of failing financial institutions on the portfolios of creditors (lenders in the model) prior to Lehman's episode.

7.1   Bailout uncertainty, the banking spread and the credit spread

The banking (or intermediation) spread.is defined as the difference between the rate, r_B, that financial intermediaries charge to borrowers and the rate, r_L, that they pay to lenders. By changing the general equilibrium level of interest rates a change in bailout uncertainty also changes the banking spread. The following proposition links the spread to bailout uncertainty.

Proposition 13:

(i) Generally, depending on whether $\frac{\partial L_F^*}{\partial r_B}$ is smaller or larger than $\mid \frac{\partial L_F^*}{\partial r_L} \mid$, an increase in bailout uncertainty raises or reduces the banking spread, $r_{B2} - r_{L2}$.

(ii) For the specification of intermediaries' objective function featured in this paper an increase in bailout uncertainty raises the banking spread, $r_{B2} - r_{L2}$.

The credit spread is generally defined as the difference between the yield on private debt instruments and government securities Within the analytical frameork of this paper the credit spread

corresponds to the difference between the rate, r_{L2}, charged by lenders to financial intermediaries and the risk free rate, r_f. The following corollary summarizes the impact of a, period's 1, increase in bailout uncertainty on the credit spread.

Corollary to proposition 12: *An increase in uncertainty about governmental bailout policy leads to an increase in the credit spread.*

Interestingly, recent empirical proxies of credit spreads due to Gilchrist and Zakrajsek (2011) show that credit spreads rose substantially during the 2007-2009 period. In particular, the substantial increase in CDS spreads in the immediate aftermath of Lehman's collapse in September 2008 is consistent with the view that those increases were due, in large part, to an increase in bailout uncertainty.

8 Exante bailout perceptions and moral hazard

8.1 The impact of lower bailout uncertainty (higher π_0)

Unlike period 1 in which the demand for credit by borrowers is already predetermined, period's 0 leverage depends on the borrowing rate in that period as well as on the borrowing rate expected to prevail in period 1. In general equilibrium both of those rates, as well as the rates at which financial intermediaries borrow from lenders, depend on financial markets participants' perceptions about the likelihood of bailout as summarized by their multiple priors set. Hence, by affecting equilibrium interest rates, perceptions about the likelihood of bailout in period 0 affect the volume of leverage in financial markets. This section investigates the impact of period's 0 permanent beliefs about governmental bailout policy as summarized by the parameter π_0 on the volume and the cost of period's 0 loans to Bs as well as on expected future rates. By affecting interest levels expected to prevail in period 1 the period's 0 perceived levels of bailout uncertainty and of the aversion to this uncertainty affect, in turn, the volume of Bs leverage in period 0. The following proposition presents the various impacts of π_0 given, overly restrictive, sufficient conditions.

Proposition 14: *For model consistent expectations and provided Fs risk aversion, δ, is small, $P_F < 1$ and $(1 - \theta_{UB})^2 (1 + r_{B1} - r_{L1}) > 1$, higher permanent values of π_0 (lower levels of bailout*

uncertainty and/or of uncertainty aversion) are associated with (i) Overall larger levels of credit by intermediaries to borrowers.and by lenders to financial intermediaries in period 0.

(ii) Lower levels of r_{B2}^e and of r_{L2}^e.

(iii) A higher level of r_{B1}

The results of Proposition 14 arise through several interconnected channels. Proposition 8-10 imply that perceptions of a more generous bailout policy directly raises the fraction of their portfolio that lenders are expected to invest in risky loans to financial intermediaries in period 1. This effect exerts **downward** pressures on the expected future rates, r_{B2}^e and r_{L2}^e. Since

$$L_B^* = \frac{1}{(1 + r_{B1})(1 + r_{B2}^e) - 1} \tag{37}$$

this **raises**, given r_{B1}, the demand for leverage by Bs. This higher demand for leverage raises r_{B1} and this **reduces** the demand for leverage by Bs. However, as suggested by proposition 14 the first effect always dominates implying that, ultimately, lower exante bailout uncertainty perceived for period 1 induces a credit expansion already in period 0.

Clearly, the belief that government may repay the debt of some delinquent financial intermediaries creates a moral hazard problem. An important implication of Proposition 14 is that, by raising leverage in the economy, the perception of a more generous bailout policy, aggravates this problem and increases the likelihood and the severity of a potential financial crisis in period 1. Note that this moral hazard problem is more important either when individuals are more confident about their probability assessments (lower ambiguity) or when they are less averse to uncertainty (lower ambiguity aversion).

8.2 The impact of a temporary expansionary monetary policy

Within the context of the model a temporary expansionary monetary in period 0, policy takes the form of a decrease in r_f holding the borrowing rate expected for period 1, r_{B2}^e, constant. The following proposition summarizes the impact of such a policy.

Proposition 15: *A temporary decrease in the risk free policy rate, r_{f1}, leads to a decrease in both r_{B1} and r_{L1}, and to an increase in leverage within both the financial and the real sectors (both $L_F^*(r_{B1}, r_{L1})$*

and L_B^* go up).

When the period's 0 decrease in r_f is permanent in the sense that it is expected to last also through period 1 there is a further expansionary effect on the equilibrium volume of credit in period 0. This effect operates through the same channels that an increase in π_0 does. That is, by reducing expected future rates, a lower expected future policy rate induces further increases in the current volume of credit.

Broad interpretation of propositions 14 and 15: The subprime crisis counterpart of period 0 in the model can be thought of as the buildup phase of the crisis. During this phase market participants believed that the set of bailout probabilities with positive mass is concentrated in a range with relatively high values of p and were relatively insensitive to the consequent uncertainty. In addition, monetary policy was loose by historical standards. Propositions 14 and 15 imply that both factors contributed to the exante expansion of credit and to a real investment boom, making the system more fragile to a sudden downward revision of perceptions about the likelihood of governmental bailouts.[30]

9   Should government commit to a bailout policy?

This section briefly reflects on the desirability (or undesirability) of bailout uncertainty. The discussion in the previous section suggests that higher **exante** bailout uncertainty and/or aversion to it reduces the volume of leverage and with it the level of real investment activity undertaken by borrowers. The discussion in section 7 suggests that higher **expost** (after long term investment decisions have been made) bailout uncertainty and/or aversion to it lead to higher levels of defaults and, in parallel, to the destruction of existing investments by borrowers. The first result opens the door for the conclusion that some exante bailout uncertainty and/or aversion to it may be desirable since it keeps the buildup of credit in check thereby alleviating potential future burdens on taxpayers as well

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[30]It is noteworthy that within four to six months after the panic that followed Lehman's collapse CDS spreads returned to subsantially lower levels. In terms of the model this is consistent with the view that, after the TARP legislation was finally passed in early October 2008 and actually implemented in the following months along with a dramatic expansion of monetary policy, bailout uncertainty along with aversion to it receded in the sense that the worst bailout probability with positive mass increased relatively to its level during the second half of September 2008. The broader implication of this observation is that government actions can reduce as well as increase both bailout uncertainty and the public's attitudes to this uncertainty.

as potential defaults. But, in the spirit of the "lender of last resort" view, the second result provides an argument for reducing bailout uncertainty and the (possibly associated aversion to it) expost in order to avoid bankrupcies and the associated destruction of capital and economic activity It would therefore appear at first blush that, although higher bailout uncertainty is disruptive expost, it may be desirable exante.

This begs the question of whether the optimal level of bailout uncertainty (or ambiguity) may be internal. Although answering this important question is largely beyond the scope of this paper it is briefly touched upon at the end of this section. But the analysis in the two previous sections does implies that, whatever the optimal level of bailout uncertainty, over time changes in both directions in this level are inefficient. This statement is based on the view that welfare in the economy is higher the lower is the volume of investments destroyed or missed due to errors in evaluating bailout uncertainty. As shown in the previous two sections an increase in bailout uncertainty and/or aversion to it between periods 0 and 1 leads to period's 0 investment and leverage levels in excess of what those levels would have been in the absence of the change, and consequently to defaults and capital destruction in period 1 in excess of their levels in the absence of the change. Hence, given the initial levels of bailout uncertainty and of aversion to it, welfare is higher in the absence of the change. When bailout uncertainty goes down period's 0 investments by borrowers are lower than what they would have been, had they known in advance that period's 1 bailout uncertainty will be lower and the level of investments in the economy is suboptimally low.

We now come back to the question regarding the desirability of bailout uncertainty. One way to reduce this uncertainty is to have government commit to a particular probability of bailout, p_c. Such a policy, if successful, eliminates multiple priors beliefs from the minds of individuals and, in parallel, reduces the impact of uncertainty aversion. Provided p_c is not higher than the minimal multiple priors entertained by individuals prior to the commitment this has the beneficial effect of minimizing excessive leverage buildups.

10   Concluding remarks

A major result of our analysis is that the larger the change in bailout uncertainty, and the potentially associated change in aversion to this uncertainty, the stronger the pre-crisis buildup and the deeper the ensuing crisis. The detailed mechanics of this result can be appreciated by thinking of period 0 as the pre-crisis phase during which the lowest perceived likelihood of bailout is high and monetary policy relatively loose leading to credit expansion and to an investment boom. Taylor (2009) argues that loose monetary policy caused, prolonged and worsened the financial crisis. Period 1 can be thought of as the phase in which, due to the arrival of some major public signal — like not rescuing Lehman — financial market operators adjust their worst scenario perceptions about the likelihood of bailout downward. By Proposition 12 this adjustment induces a general increase in market interest rates, a rise in the proportion of insolvent borrowers along with the destruction of real investments and, for some realizations of real returns, a complete drying up of short term credit markets – or in Caballero's (2010) terminology – a sudden financial arrest.

By Proposition 14 the pre-crisis bubbly credit boom is larger the larger π_0. By Proposition 12 the magnitudes of deleveraging and of insolvencies (real and financial) is larger the lower is π_1. Since it measures the extent to which the subjective set of possible bailout distributions widened between periods 0 and 1 the difference $\pi_0 - \pi_1$ is a natural proxy for the increase in bailout uncertainty cum aversion. Combining this proxy with Propositions 14 and 12 yields the conclusion that higher changes in bailout uncertainty cum uncertainty aversion are associated with larger pre-crisis bubbles as well as with higher levels of insolvencies and destruction of real economic activity when the bubble bursts. The crucial variable through which those effects operate is leverage. It expands more during periods of optimism about the likelihood of bailouts but, by the same token, it shrinks more violently during periods of pessimism about the likelihood of bailouts.[31] Given π_1 the deleveraging process during period 1 involves a larger volume of insolvencies the larger is π_0. The reason is that a larger π_0 raises the exante leverage buildup in comparison to what market operators would have engaged

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[31]In terms of the multiple priors model periods of optimism can be thought as being characterized by low ambiguity aversion and period of pessimism by high levels of ambiguity aversion.

in, had they known already in period 0 that the probability of bailout in period 1 will drop to π_1. The larger this "excessive" credit buildup, the larger the expost volume of insolvencies in the real economy.

The paper shows that the perceptions of market participants about systemic risks as captured by the aggregate covariance between the returns on different loans within the fully diversified portfolio of lenders is systematically related to bailout uncertainty and ambiguity aversion. In particular, even within a fully rational world, a higher level of bailout uncertainty and/or uncertainty aversion is associated with a higher level of perceived systemic risks (propositions 10 and 12). A novel broader implication of this analysis is that, in the presence of bailout uncertainty, the level of systemic risk may increase just because of an increase in ambiguity aversion following a traumatic event such as Lehman's collapse implying that systemic risk levels depend on preferences. Furthermore, the effect of increasing systemic risk due to changes in preferences for ambiguity is likely to be persistent.

Somewhat analogously to Diamond and Dybvig (1983) (DD in the sequel) classic model of bank runs, a main objective of this paper was to identify circumstances that trigger a financial crisis. A main result of the DD framework is that deposit insurance eliminates runs on the banks. Although there is an analogy between the role of deposit insurance in DD and bailouts in our framework, a crucial difference between them is that, up to a given limit, deposit insurance is backed by the exante certainty of a legal act while the availability (and scope) of the generalized bailouts considered here is shrouded in uncertainty and is likely to remain in this state also in the future. Besides other obvious differences two additional difference worth emphasizing are: (i) In DD liquidity shocks are exogenous while here they are related to an increase in uncertainty due to the arrival of new information about "black swan" events. (ii) Our framework features a more detailed picture of the financial sector and is designed to make statements about the impact of monetary policy on leverage and the economy. Like Caballero and Krishnamurthy (2008) this paper attributes flight to quality episodes to the existence of Knightian uncertainty. But we argue that flight to quality episodes are likely to additionally reflect increases in aversion to uncertainty following traumatic, largely unanticipated events like the downfall of Lehman's Brothers. One implication of this view is that, such events tend to raise both

uncertainty and the aversion to it. Although, in practice, individuals may find it hard to disentangle uncertainty from uncertainty aversion, the Gilboa-Schmeidler (1989) max-min with multiple priors framework utilized in the paper does not require them to seperate between those factors in order to optimally react to such changes. More broadly decreases in uncertainty and in uncertainty aversion provide a structured way to understand some of the factors that lie behind Keynes "animal spirits" and to link them to abrupt changes in government policy and other exogenous events.

Reinhart and Rogoff (2009) present broad evidence supporting the view that private financial crises are often followed by substantial reductions in tax collections and defaults on sovereign debt. Motivated by this findings and some of the results in this paper we speculate in what follows on an additional channel through which higher exante leverage buildups possibly makes the economy more crisis prone when new information arrives. Higher leverage raises the probability as well as the magnitude of potential defaults, and with it the cost of potential bailouts. The more costly is a bailout to taxpayers the more reluctant is government to engage in such bailouts. As a consequence, the higher is leverage, the higher bailout uncertainty making beliefs more sensitive to news.

The upshot is that the sensitivity of expectations to various news becomes larger the larger is leverage. In terms of the multiple priors framework this means that the range of bailout probability distributions entertained by individuals becomes more sensitive to news. As a consequence, the same pessimistic new information about the likelihood of bailout is more likely to puncture a bubble the higher is leverage.

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A   Appendix

Proof of Lemma 1: For regular borrowers, and given $r_{B2} = r_{B2}^e$, the condition in Equation (4) is identical to the condition in Equation (3). But, by assumption 1a, the last condition is satisfied for all $L_B \geq 0$. Hence, regular borrowers are solvent at any level of leverage. Since $2yq > 2y(2q-1)$ this is afortiori true for lucky borrowers. The proof for unlucky borrowers follows by using Equation (5) in solvency condition (4) and by rearranging. QED

Proof of Lemma 2: The proof of parts (i) and (ii) is obtained by substituting $\widetilde{Y}_P = -2y$ and $\widetilde{Y}_P = 0$ respectively into Equation (8) and by rearranging. The proof of part (iii) follows by inserting $\widetilde{Y}_P = 2y$ into Equation (8) and by utilizing Assumption 1. QED

Proof of Lemma 3: Part (i) is a direct consequence of Lemma 1. To prove part (ii) note that, since $\frac{1}{2} < q < 1$, $L_B^L < L_B^1 < L_B^H$. The ultimate payoff of an UB is either 0 in case of expansion or $-2y$ in case of contraction. By Lemma 2, and since $L_B^1 < L_B^H$, this borrower is solvent in the first case. By Lemma 2, and since $L_B^L < L_B^1$, this borrower is insolvent in the second case. Part (iii) is a direct consequence of Assumption 1 in conjunction with condition (7). QED

Proof of Proposition 1: The first two default probabilities follow directly from Lemmas 1 through 3. The third probability is derived by noting that, when leverage is larger than L_B^1 default may occur in period 1 if the borrower turns out to be unlucky (probability θ_{UB}) and may also occur in period 2 if he turns out to be a regular borrower and $\widetilde{Y}_P = -2y$ (probability $(1-\theta)(1-q)^2$). The last probability follows by noting that, in addition to the states in which he defaults in the previous case, the borrower defaults also in the following two cases: (i) If he is a LB and there is a contraction, (ii) If he is a regular borrower in and $\widetilde{Y}_P = 0$. QED

Proof of Proposition 2: To show that L_B^H is the optimal level of leverage it suffices to establish that

$$V(L_B^H) > V(L_B^1) > V(L_B^L) > V(L_B = 0),$$

$$V(L_B^H) > V(L_B^m).$$

The proof is implemented by using Equation (12) to form explicit expressions for the differences

$V(L_B^L) - V(L_B = 0)$, $V(L_B^1) - V(L_B^L)$, $V(L_B^H) - V(L_B^1)$, $V(L_B^m) - V(L_B^H)$ and by showing that they are all positive.

(i) $V(L_B^L) - V(L_B = 0) = q^2(2y - r_B^e)L_B^L - 2q(1-q)r_B^e L_B^L$.

Assumption 1 and q sufficiently close to one imply that this difference is positive.

(ii) $V(L_B^1) - V(L_B^L) = \{q^2(2y - r_B^e) - 2q(1-q)r_B^e\}(L_B^1 - L_B^L) - (1-q)^2 P_B L_B^1$.

Since $(L_B^1 - L_B^L) > 0$ this expression is positive for q sufficiently close to one.

(iii) Letting θ_{LB} approach q from below and θ_{UB} approach zero from above

$$V(L_B^H) - V(L_B^1) = q^2(2y - r_B^e)(L_B^H - L_B^1) - 2q(1-q)(1 - r_B^e L_B^1) - (1-q)^2 P_B \left[(1-q)L_B^H - L_B^1\right].$$

Since $(L_B^H - L_B^1) > 0$ this expression is positive for q sufficiently close to one.

(iv) The condition $V(L_B^m) - V(L_B^H) > 0$ is equivalent to

$$[[(1-\theta)2q(1-q) + \theta_{LB}(1-q)] P_B - [\theta_{LB}q + (1-\theta)q^2](2y - r_B^e)] (L_B^m - L_B^H) > 0.$$

Since $(L_B^H - L_B^1) > 0$ this expression is positive if and only if $P_B > P_B^c$. QED

Proof of Proposition 3: The proof is a direct consequence of the fact that all three types of agents are risk averse and that leverage levels are positive. Consequently, financial intermediaries require a markup over their leverage costs as compensation for investing in risky loans to investors. As a consequence $r_B > r_L$. Similarly, lenders demand a risk premium when they invest in risky loans to financial intermediaries rather than in risk free asset. Hence, $r_L \geq r_f$.[32] QED

Proof of Proposition 4: By construction, since $r_f < r_L$, an F with positive short term leverage and some fraction of the portfolio invested in risk free assets can increase his profits by reducing both short term leverage and, in parallel, the investment in risk free assets. Consequently, a configuration with both positive leverage and some investment in risk free assets cannot be a financial intermediary's optimum. Hence $z_f = 0$. QED

Proof of Proposition 5: Equation (19) is obtained by maximizing the expected utility of F with respect to L_F for $z_f = 0$ under the assumption that F's optimal leverage is above L_F^c. Conditions (ii) and (iii) are needed to rule out the possibility that optimal leverage is at L_F^c or at zero. To derive

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[32] As elaborated later in the general equilibrium section, the equality sign is allowed to accommodate a special case in which, due to full diversification, lenders face no risk in period 0.

those conditions let L_F^m be any leverage level above L_F^c and let.

$$EV_F\left[L_F\right] \equiv Eu\left[W_F(L_F)\right]. \tag{38}$$

Then necessary conditions for the optimal level of leverage to be above L_F^c are

$$EV_F\left[L_F^m\right] \;>\; EV_F\left[L_F = 0\right]$$

$$EV_F\left[L_F^m\right] \;>\; EV_F\left[L_F^c\right] \tag{39}$$

Conditions (i) and (ii) are obtained by using equations (13) and (16) to express F's utility in terms of the appropriate levels of leverage, substituting the resulting expression into Equation (39) and by rearranging. To complete the proof it remains to show that, when $(\gamma_{2t}/\gamma_{1t})$ is sufficiently small, $EV_F\left[L_F^c\right] > EV_F\left[L_F^s\right]$ for any $0 < L_F^s < L_F^c$ establishing (since $EV_F\left[L_F^m\right] > EV_F\left[L_F^c\right]$) that $EV_F\left[L_F^m\right]$ is also larger than $EV_F\left[L_F^s\right]$. When $(\gamma_{2t}/\gamma_{1t})$ is small the only two terms that could potentially make $EV_F\left[L_F^s\right]$ larger than $EV_F\left[L_F^c\right]$ involve γ_{2t}, while the terms that operate to reverse this inequality involve γ_{1t}. In the extreme case, $(\gamma_{2t}/\gamma_{1t}) = 0$, it is unambiguously the case that $EV_F\left[L_F^c\right] > EV_F\left[L_F^s\right]$. By continuity this is also the case for $(\gamma_{2t}/\gamma_{1t})$ positive but sufficiently small. QED

Proof of Proposition 6: The condition in Equation (17) implies that, when $L_{Ft}^* > L_F^c$, F is solvent if and only if both of his debtors are solvent. Recalling that $r_{Bt} - r_{Lt} > 0$ and inspecting Equation (19) reveals that L_{Ft}^* is a monotonically increasing function of δ and that it tends to infinity when δ tends to zero. It follows that, for sufficiently small but positive values of δ, $L_{Ft}^* > L_F^c$ implying that F is solvent if and only if both of his borrowers are solvent. QED

Proof of Proposition 7: The first three parts follow directly from inspection of Equation (19). Part (iv) is established by differentiating this equation with respect to r_{Bt}. QED

Proof of Proposition 8: Calculation of the expected value is relatively straightforward. Derivation of the variance utilizes the fact that the variance of a fully diversified risky portfolio composed of (equally weighted) infinitely many identically distributed assets is equal to the covariance between any two assets within the portfolio. Calculation of this covariance simplifies the derivation of an explicit expression for $Var\left(\{\widetilde{R}_{L2}\}\right)$ but still involves some messy intermediate algebra. The expression

for L's portfolio variance in Proposition 8 is obtained by using the joint distribution of $(\tilde{r}_{Li}, \tilde{r}_{Lj})$ in the definition of the covariance between $\tilde{r}_{Li}$ and $\tilde{r}_{Lj}$ and by simplifying the resulting algebraic expressions.. QED

Proof of Proposition 9: Recall that, to keep notation simple, we use the symbol $\tilde{R}_L$ to denote the gross (one plus) return on a portfolio that consist of an infinite number of loans, $\{\tilde{r}_L\}$, and $\tilde{R}_f = 1 + r_f$ is the gross risk free rate. A typical lender's maximization problem is given by

$$\max_{z_L} E\left[u\left(z_L(\tilde{R}_L - R_f) + R_f\right)\right],$$

where $u(\cdot)$ stands for the utility function. The first order condition implies

$$E\left[u'\left(z_L^*(\tilde{R}_L - R_f) + R_f\right)(\tilde{R}_L - R_f)\right] = 0.$$

Taking a Taylor approximation of the marginal utility with respect to $\tilde{R}_L$ around R_f yields

$$E\left[u'(R_f)\left(\tilde{R}_L - R_f\right)\right] + E\left[u''(R_f)\left(\tilde{R}_L - R_f\right)\left(\tilde{R}_L - R_f\right)z_L^*\right] \cong 0.$$

For a sufficiently small risk premium

$$z_L^* \cong -\frac{u'(R_f)\left(E\tilde{R}_L - R_f\right)}{u''(R_f) E\left[\left(\tilde{R}_L - R_f\right)^2\right]} = \frac{\left(E\tilde{R}_L - R_f\right)}{-\frac{u''(R_f)}{u'(R_f)}Var\left(\tilde{R}_L\right)},$$

but for constant absolute risk aversion, $u(x) = -e^{-\alpha x}$, the coefficient of absolute risk aversion is $-\frac{u''(R_f)}{u'(R_f)} = \alpha$, and thus $z_L^* \cong \frac{E(\{1+\tilde{r}_L\}) - (1+r_f)}{\alpha Var(\{\tilde{r}_L\})}$. QED

Proof of Proposition 10: Part (i) follows immediately form Equation (22).

Part (ii) The first derivative of $Var\left(\{\tilde{R}_{L2}\}\right)$ with respect to p is

$$\frac{\partial Var\left(\{\tilde{R}_{L2}\}\right)}{\partial q} = -2(1-p)(1-q)(1-q_1)^2\left(q + 2qq_1 + q_1^2\right),$$

which is negative since $p, q \in [0, 1]$ are probabilities.

Part (iii) follows immediately form Equation (22). QED

Proof of Proposition 11: Writing Equation (27) as

$$z_L^*(\pi, r_L, q, q_1) \cong \frac{E\left(\{\tilde{R}_L\}\right) - (1 + r_f)}{\alpha Var\left(\{\tilde{R}_L\}\right)} = \frac{\lambda_1(1 + \tilde{r}_L) - (1 + r_f)}{\alpha \lambda_2(1 + \tilde{r}_L)^2} = \frac{1}{\alpha \lambda_2}y,$$

where λ_1 and λ_2 are defined by equations (22) and (23), respectively. Differentiating y with respect to $\widetilde{r}_L$ yields

$$\frac{\partial y}{\partial \widetilde{r}_L} = \frac{-\lambda_1 \left(1 + \widetilde{r}_L\right) + 2\left(1 + r_f\right)}{\left(1 + \widetilde{r}_L\right)^3}.$$

Since it is a probability $\lambda_1 \in [0,,1]$. Hence, the derivative is positive for $\frac{2\left(1+r_f\right)-\lambda_1}{\lambda_1} \geq 1 + 2r_f \geq \widetilde{r}_L$.

QED

Proof of Proposition 12 : Part (i) is an immediate consequence of proposition 10.

Part (ii): Differentiating equations (33) and (34) totally with respect to π yields

$$0 = (1 - \theta_{UB})M_F \left[\frac{\partial L_F^*}{\partial r_B}\frac{dr_{B2}}{d\pi} + \frac{\partial L_F^*}{\partial r_L}\frac{dr_{L2}}{d\pi}\right], \tag{40}$$

$$(1 - \theta_{UB})^2 M_F \left[\frac{\partial L_{F2}^*}{\partial r_B}\frac{dr_{B2}}{d\pi} + \frac{\partial L_{F2}^*}{\partial r_L}\frac{dr_{L2}}{d\pi}\right] = (1 + r_{L1})M_L \left[\frac{\partial z_{L2}^*}{\partial r_L}\frac{dr_{B2}}{d\pi} + \frac{\partial z_{L2}^*}{\partial \pi}\right]. \tag{41}$$

Solving this two equations system for $\frac{dr_{B2}}{d\pi}$ and $\frac{dr_{L2}}{d\pi}$

$$\frac{dr_{B2}}{d\pi} = -\frac{\frac{\partial L_{F2}^*}{\partial r_L}\frac{\partial z_{L2}^*}{\partial \pi}}{\frac{\partial L_{F2}^*}{\partial r_B}\frac{\partial z_{L2}^*}{\partial r_L}}, \tag{42}$$

$$\frac{dr_{L2}}{d\pi} = -\frac{\frac{\partial z_{L2}^*}{\partial \pi}}{\frac{\partial z_{L2}^*}{\partial r_L}}. \tag{43}$$

By Proposition 7, $\frac{\partial L_{L2}^*}{\partial r_B} > 0$ and $\frac{\partial L_{L2}^*}{\partial r_L} < 0$. By Proposition 10 and 11, $\frac{\partial z_{L2}^*}{\partial \pi}$ and $\frac{\partial z_{L2}^*}{\partial r_L}$ are both positive. Utilization of those sign restrictions in equations (42) and (43) implies that the general equilibrium effects of a surprise decrease in π is to raise both r_{B2} and r_{L2} above what those rates had been expected to be in period 0 (r_{B2}^e and r_{L2}^e).

Part (iii): Although Assumption 2 requires that $E\left[\widetilde{Y}_P \mid I_1 \cap RB\right] = 2y([q + q_1 - 1] > r_B^e \equiv (1 + r_{B1})(1 + r_{B2}^e) - 1$ the condition in part (ii) of this proposition implies that it is violated when r_B^e is replaced with r_B, or in explicit notation

$$E\left[\widetilde{Y}_P \mid I_1 \cap RB\right] = 2y([q + q_1 - 1] < r_B \equiv (1 + r_{B1})(1 + r_{B2}) - 1. \tag{44}$$

By equation (4) adapted to **actual** period's 1 information a RB is solvent in period 1 if and only if

$$L_B(1 + r_B^e) \leq (1 + L_B)\left[1 + E\left[\widetilde{Y}_P \mid I_1 \cap RB\right]\right],$$

which is equivalent to

$$L_B \leq \frac{1 + 2y([q + q_1 - 1]}{r_B - 2y([q + q_1 - 1]} \equiv L_B^1(r_{B1}), \tag{45}$$

where the denominator is positive by condition (44). It follows that regular borrowers do not get refinancing in period 1 if

$$L_B^* = L_B^H = \frac{1}{r_B^e} > \frac{1 + 2y([q + q_1 - 1]}{r_B - 2y([q + q_1 - 1]} \equiv L_B^1(r_{B1}). \tag{46}$$

Rearrangement of this inequality reveals that it is equivalent to the condition in part (ii) of the proposition establishing that RB default. Given that RB default under r_{B2} UB default afortiori.

Part (iv) When r_{B2} increases sufficiently beyond the bound in part (ii) even LB are denied access to credit inducing a total drying up of refinancing to borrowers. QED

Proof of proposition 13 : Part (i): The banking spread in period 2 is

$$S_2 \equiv r_{B2} - r_{L2}. \tag{47}$$

Differentiating the spread totally with respect to π_1

$$\frac{dS_2}{d\pi_1} = \frac{dr_{B2}}{d\pi_1} - \frac{dr_{L2}}{d\pi_1} = \left(1 - \frac{\mid \frac{\partial L_{F2}^*}{\partial r_L} \mid}{\frac{\partial L_{F2}^*}{\partial r_B}}\right) \mid \frac{dr_{L2}}{d\pi_1} \mid \tag{48}$$

where the second equality follows by using equations (42) and (43). Since $\frac{\partial L_{F2}^*}{\partial r_B} > 0$, $\frac{dS_2}{d\pi_1}$ is negative or positive depending on whether $\mid \frac{\partial L_{F2}^*}{\partial r_L} \mid$ is larger or smaller than $\frac{\partial L_{F2}^*}{\partial r_B}$. Hence an increase in bailout uncertainty (a decrease in π_1) raises the spread in the first case and reduces it in the second.

Part (ii): Examination of the expression for L_{F2}^* in proposition 5 reveals that $\mid \frac{\partial L_{F2}^*}{\partial r_L} \mid$ and $\frac{\partial L_{F2}^*}{\partial r_B}$ may differ only because of the second term on the right hand side of equation (19). Using this fact along with that equation yields after some algebra $\mid \frac{\partial L_{F2}^*}{\partial r_L} \mid - \frac{\partial L_{F2}^*}{\partial r_B} = \frac{r_{B2} - r_{L2}}{S_2^2}$. Since $r_{B2} - r_{L2} > 0$, $\mid \frac{\partial L_{F2}^*}{\partial r_L} \mid > \frac{\partial L_{F2}^*}{\partial r_B}$ implying, by part (i), that an increase in bailout uncertainty raises the banking spread. QED

Proof of proposition 14: The 3 equations system in (30), (31) and (32) determines r_{B2}^e, r_{L2}^e and r_{B1} as functions of π_0 and other exogenous variables. Differentiating this system totally with respect to π_0 and using the total differential of the first of those equations to express $\frac{dr_{B1}}{d\pi_0}$ in terms of $\frac{dr_{B2}}{d\pi_0}$

yields

$$\frac{dr_{B1}}{d\pi_0} = \frac{dr_{B1}}{dr^e_{B2}}\frac{dr^e_{B2}}{d\pi_0} \tag{49}$$

where

$$\frac{dr_{B1}}{dr^e_{B2}} = -\frac{M_B\frac{\partial L^*_B}{\partial r^e_B}}{M_B\frac{\partial L^*_B}{\partial r_B} - M_F\frac{\partial L^*_{F1}}{\partial r_B}}. \tag{50}$$

Equation (37) implies $\frac{\partial L^*_B}{\partial r^e_B} < 0$, $\frac{\partial L^*_B}{\partial r_B} < 0$ and, from proposition 7 $\frac{\partial L^*_{F1}}{\partial r_B} > 0$. Hence $\frac{dr_{B1}}{dr^e_{B2}}$ is negative implying that $\frac{dr_{B1}}{d\pi_0}$ and $\frac{dr_{B2}}{d\pi_0}$ have opposite signs. Differentiating L^*_B totally with respect to π_0 and using equation (50)

$$\frac{dL^*_B}{d\pi_0} = \left\{ -\frac{M_F\frac{\partial L^*_{F1}}{\partial r_B}\frac{\partial L^*_B}{\partial r^e_B}}{M_B\frac{\partial L^*_B}{\partial r_B} - M_F\frac{\partial L^*_{F1}}{\partial r_B}} \right\}\frac{dr^e_{B2}}{d\pi_0}. \tag{51}$$

Inspection reveals that the term in curly brackets on the right hand side of this equation is negative implying that

$$\mathrm{Si}\,gn\left(\frac{dL^*_B}{d\pi_0}\right) = -\,\mathrm{Si}\,gn\left(\frac{dr^e_{B2}}{d\pi_0}\right) \tag{52}$$

Substituting equation (50) into the total differentials of equations (31) and (32) and rearranging yields the following two equations system for the determination of $\frac{dr^e_{B2}}{d\pi_0}$ and of $\frac{dr^e_{L2}}{d\pi_0}$.

$$M_F(1-\theta_{UB})^2\frac{\partial L^e_{F2}}{\partial r^e_B}\frac{dr^e_{B2}}{d\pi_0} + \left(M_F(1-\theta_{UB})^2\frac{\partial L^e_{F2}}{\partial r^e_L} - M_L(1+r_{f1})\frac{\partial z^e_{L2}}{\partial r^e_L}\right)\frac{dr^e_{L2}}{d\pi_0} = M_L(1+r_{f1})\frac{\partial z^e_{L2}}{\partial \pi_0}$$

$$\left\{ \begin{array}{c} M_F(1-\theta_{UB})\left(W_{r_{B1}}\frac{dr_{B1}}{dr^e_{B2}} + \frac{\partial L^e_{F2}}{\partial r^e_B}\right) \\ -M_B\left(\frac{\partial L^*_B}{\partial r_B}\frac{dr_{B1}}{dr^e_{B2}} + \frac{\partial L^*_B}{\partial r^e_B}\right) \end{array} \right\}\frac{dr^e_{B2}}{d\pi_0} + M_F(1-\theta_{UB})\frac{\partial L^e_{F2}}{\partial r^e_L}\frac{dr^e_{L2}}{d\pi_0} = 0 \tag{53}$$

where L^e_{F2} and z^e_{L2} are the model consistent expectations of those variables given the information set of period 1 and $W_{r_{B1}}$ is the derivative of $\{1 + r_{B1} + (r_{B1} - r_{L1})L^*_F(r_{B1}, r_{L1})\}$ with respect to r_{B1}. To evaluate the signs of $\frac{dr^e_{B2}}{d\pi_0}$ and of $\frac{dr^e_{L2}}{d\pi_0}$ it is convenient to utilize the following claim that is proved later. QED

Claim: Provided δ is positive but close to zero, $P_F < 1$ and $(1 - \theta_{UB})^2(1 + r_{B1} - r_{L1}) > 1$ the impact of a general equilibrium change in r^e_{B2} on r_{B1} (i.e. $\frac{dr_{B1}}{dr^e_{B2}}$) is relatively small.

Using the claim in equations (53) and solving for $\frac{dr^e_{B2}}{d\pi_0}$ and $\frac{dr^e_{L2}}{d\pi_0}$

$$\frac{dr_{B2}^e}{d\pi_0} \cong \frac{\frac{\partial L_{F2}^e}{\partial r_L^e}\frac{\partial z_{L2}^e}{\partial \pi_0}}{\frac{\partial z_{L2}^e}{\partial r_L^e}\frac{\partial L_{F2}^e}{\partial r_B^e}} < 0$$

$$\frac{dr_{L2}^e}{d\pi_0} \cong -\frac{\left(M_F(1-\theta_{UB})\frac{\partial L_{F2}^e}{\partial r_B^e} - M_B\frac{\partial L_B^*}{\partial r_B^e}\right)\frac{\partial z_{L2}^e}{\partial \pi_0}}{M_F(1-\theta_{UB})\frac{\partial z_{L2}^e}{\partial r_L^e}\frac{\partial L_{F2}^e}{\partial r_B^e}} < 0. \tag{54}$$

The negative signs of those expressions follow by noting that, from proposition 7, $\frac{\partial L_{F2}^e}{\partial r_B^e} > 0$, $\frac{\partial L_{F2}^e}{\partial r_L^e} < 0$; from propositions 10 and 11 $\frac{\partial z_{L2}^e}{\partial \pi_0}$ and $\frac{\partial z_{L2}^e}{\partial r_L^e}$ are both positive; and from equation (37) $\frac{\partial L_B^*}{\partial r_B^e} < 0$. This establishes part (ii). The fact that L_B^* is higher follows from equation (52) in conjunction with the fact that $\frac{dr_{B2}^e}{d\pi_0} < 0$. This implies via equations (31) and (30) that $L_F^*(r_{B1}, r_{L1})$ and z_{L1} are also higher when π_0 is higher establishing part (i). Part (iii) follows from part (ii) and equation (49) by noting that $\frac{dr_{B1}}{dr_{B2}^e}$ is negative. This completes the proof of the proposition provided the claim is true.

Proof of claim: Noting that $\gamma_{11} = (1-\theta_{UB})^2$ and differentiating equation (19) with respect to r_{B1}

$$\frac{\partial L_{F1}^*}{\partial r_B} = \frac{1-\delta}{\delta}\left\{\frac{1}{P_F}\frac{(1-\theta_{UB})^2}{1-(1-\theta_{UB})^2}(r_{B1}-r_{L1})^{1-2\delta}\right\}^{\frac{1}{\delta}} + \frac{1+r_{L1}}{(r_{B1}-r_{L1})^2}.$$

Under the conditions of the proposition the term in curly parenthesis on the right hand side of this equation is larger than one. Consequently, when δ tends to zero from above $\frac{\partial L_{F1}^*}{\partial r_B}$ tends to infinity implying, by equation (50), that $\frac{dr_{B1}}{dr_{B2}^e}$ becomes very small. This completes the proof of the claim and of the proposition. QED

Proof of Proposition 15: Translated into the model's timing framework a temporary decrease in the risk free rate means that r_{f1} goes down without any change in r_{f2}^e. The decrease in r_{f1} triggers, via equation (28), a decrease in r_{L1}, and by part (ii) of proposition 7, an increase in $L_F^*(r_{B1}, r_{L1})$. This increase translates through equilibrium condition (29) into an increase in z_{L1}. Furthermore, the increase in $L_F^*(r_{B1}, r_{L1})$ induces, via equilibrium condition (30), a decrease in r_{B1} and an increase in L_B^*. QED